MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. JUNE 8, 1917. No. 8

Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Cyril Heath Stephenson, Bickford House, Fallings Park, Wolverhampton (proposed by Arthur Hands); and

William Wilson, M.B., C.M., Medical Missionary of the China Inland Mission, 43 Fellows Road, South Hampstead, N.W. 3 (proposed by A. M. Newbegin).

Professor C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, Washington, D.C., U.S.A.;

Professor Giovanni Celoria, Director, Reale Osservatorio di Brera, Milan, Italy;

M. Pierre Henri Puiseux, Observatoire, Paris; and

Professor Charles Edward St. John, F.R.A.S., Solar Observatory, Mount Wilson, California, U.S.A.,

were balloted for and duly elected Associates of the Society.

Forty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Enlarged photograph of parts of two letters from Lalande to Sir W. Herschel, suggesting a symbol for the planet Uranus, presented by Miss Francisca Herschel.

SPECIAL GENERAL MEETING.

A Special General Meeting, of which due notice had been given to the Fellows, was held the same day, at 6.45 o'clock. Sir F. W. Dyson, M.A., F.R.S., Vice-President, in the Chair.

The following new By-law, proposed by the Council, was put to the meeting and carried :—

BY-LAW SPECIAL TO THE PRESENT WAR.

26A. Notwithstanding the preceding By-laws, any Fellow of the Society who is on Active Naval or Military Service may, if he so desires, be excused from payment of his Annual Contribution until the end of the "emergency period" (*i.e.* the end of the year following that in which peace is declared). He shall not during that period forfeit his Fellowship, and his name shall not be suspended as a defaulter by reason of non-payment of such Annual Contribution, as provided in By-law 26; but he shall not receive the publications of the Society until he resumes the payment of his Annual Contribution.

At the end of the emergency period he shall have the option of resuming his Annual Subscription from that date, or any previous date, receiving the publications of the Society from the date of resumption; or of resigning his Fellowship without any liability for arrears of Subscription.

Note on the Paper "On Frederick de Houtman's Catalogue of Stars." By E. B. Knobel.

I am indebted to Dr. F. C. Wieder of Wageningen, and to Dr. J. Stein, S.J., of Amsterdam, for calling my attention to the fact that the map of Plancius, dated 1594, mentioned on p. 414, was preceded by a map published by Plancius in 1592, of which only one copy is known to exist, and that at Valencia in Spain. This map of 1592 is referred to by Blundeville in his *Description of Petrus Plancius, his universall Mappe*, and is thus clearly the origin of Blundeville's remarks quoted on p. 416.

I also learn that two further copies of Houtman's Catalogue are known; one in the Royal Library at the Hague, and the other in a private library in Holland.

The Systematic Motions of the Stars between Dec. + 24° and Dec. + 32°. By Sir F. W. Dyson and W. G. Thackeray. (Plates 8 and 9.)

The proper motions discussed in this paper are those which have been determined for the Greenwich Catalogue for 1910·0, and are about 12,000 in number. They were obtained by comparison of the Greenwich positions with those given in the catalogues of the Astronomische Gesellschaft, supplemented by determinations from the earlier catalogues of Bessel and Lalande. In the reduction of Bessel's and Lalande's observations corrections were found for each separate night's observations. The probable error of the proper motions varies considerably. It may be put at $\pm 1''{\cdot}25$ a century, except for the Right Ascensions of about 2500 stars which are derived from a comparison of Greenwich and Cambridge alone. For these it is larger, say $\pm 1''{\cdot}50$ a century.

Each hour of Right Ascension has been considered separately, the zone being thus divided into 24 areas of approximately $13^\circ \times 8^\circ$. The proper motion of every star in each of these areas has been plotted just as if the star were situated at the centre of the area in question. The plots were made from the proper motions in Right Ascension (converted into arc) and Declination, but by making them on paper ruled with circles and radiating lines the composition into total proper motions and position-angles was made graphically. The zero of angle is the direction of increasing Right Ascension, and the direction of increasing angle is clockwise, so that 90° indicates a proper motion towards the North Pole.

From these plots can be read off by inspection either the number of stars between given limits of direction or the number of stars whose total proper motions lie between certain limits. For convenience, the stars in each hour were divided into two groups, those from 0″ to 5″ being plotted on one sheet of paper and those above 5″ on another.

Part I.

Star Streaming.

The plots bring out in a striking manner the want of symmetry in the direction of proper motions due to star streaming. For the larger proper motions there is a considerable sector, amounting sometimes to 180°, in which no stars appear, while in the opposite sector there is no direction of maximum concentration. With smaller proper motions these characteristics are less marked, while with the very small proper motions the effect of accidental error has produced still greater uniformity of distribution. This will be clearly seen in Table I.

TABLE I.

Number of Stars within certain Limits of Proper Motion for every 10° of Direction.

Angle of Direction.	0^h–1^h.				1^h–2^h.				2^h–3^h.			
	0″·2–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″·2–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″·2–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
°0–10	9	6	5	7	7	5	0	6	2	6	3	1
10–20	6	8	3	5	5	3	2	1	5	4	2	3
20–30	4	4	3	3	2	3	0	1	3	2	1	0
30–40	8	2	0	2	2	2	1	0	4	3	0	0
40–50	1	1	1	1	2	0	1	2	2	0	1	0
50–60	3	0	0	0	6	2	1	0	2	0	1	0
60–70	4	1	1	0	3	0	0	0	5	1	3	0
70–80	9	2	1	0	3	1	1	0	1	2	0	0
80–90	2	0	0	0	4	3	0	0	2	0	0	0
90–100	6	0	0	0	4	2	1	0	2	1	0	0
100–110	1	3	0	0	2	0	0	0	1	0	0	2
110–120	5	1	0	0	1	1	0	0	4	1	1	1
120–130	2	1	0	0	1	1	0	0	6	3	0	0
130–140	5	2	2	0	2	2	4	0	1	1	0	1
140–150	3	1	1	0	9	1	0	0	3	2	2	1
150–160	3	2	2	2	3	2	1	0	4	2	0	0
160–170	3	7	0	0	4	0	2	0	5	3	1	0
170–180	5	9	2	0	3	4	4	1	5	4	0	1
180–190	4	3	1	0	4	7	3	0	7	0	2	0
190–200	6	2	2	1	8	3	1	3	4	2	3	1
200–210	13	8	4	2	5	2	1	0	7	0	3	1
210–220	7	6	4	1	8	2	2	1	6	3	5	0
220–230	5	7	3	2	7	12	3	3	3	5	3	5
230–240	3	2	1	0	6	4	2	4	5	3	1	0
240–250	7	3	4	2	10	5	1	2	7	4	6	0
250–260	7	4	1	1	6	2	3	1	7	3	2	2
260–270	7	4	6	2	6	6	1	0	4	4	2	0
270–280	6	3	3	0	4	2	3	3	7	2	2	1
280–290	3	7	2	3	8	2	3	1	4	4	1	0
290–300	6	6	0	1	4	2	1	2	7	5	6	0
300–310	4	3	2	3	6	7	2	5	1	3	7	4
310–320	9	4	0	2	4	8	4	4	7	5	1	3
320–330	5	4	4	5	7	4	3	3	8	7	7	8
330–340	12	2	4	9	14	7	1	9	2	3	4	6
340–350	5	5	5	14	6	7	5	8	7	8	4	7
350–360	6	10	5	11	4	5	7	3	2	9	4	4

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	3^h–4^h.				4^h–5^h.				5^h–6^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
° °												
0–10	4	4	3	0	3	3	0	0	11	7	2	0
10–20	5	2	0	0	3	2	0	2	8	7	1	0
20–30	1	2	0	0	3	2	0	0	10	4	0	0
30–40	4	3	1	0	5	2	1	0	1	5	0	0
40–50	4	3	0	0	1	1	1	1	7	3	0	0
50–60	4	1	0	0	5	1	0	1	2	2	0	0
60–70	3	1	0	0	4	2	0	0	3	0	0	0
70–80	3	1	0	0	1	1	0	0	1	1	0	0
80–90	3	0	0	1	3	1	0	0	6	1	0	0
90–100	6	1	0	0	2	1	0	1	5	0	0	0
100–110	1	0	0	0	0	1	1	0	2	1	1	0
110–120	2	2	0	0	4	2	0	0	9	1	1	0
120–130	3	0	0	0	1	1	0	0	5	3	0	0
130–140	5	0	0	0	1	0	0	0	6	0	1	0
140–150	1	2	0	0	1	0	1	0	8	4	0	0
150–160	4	2	0	0	1	2	0	0	6	2	2	0
160–170	1	3	0	1	1	0	1	0	4	2	0	0
170–180	4	4	0	0	5	2	1	1	9	4	1	1
180–190	3	3	4	0	2	3	2	0	6	4	2	0
190–200	3	5	1	1	3	5	2	0	8	6	5	0
200–210	5	3	2	0	5	2	0	1	5	2	2	0
210–220	5	3	4	0	3	3	4	0	8	5	1	2
220–230	4	7	0	1	1	1	0	3	13	9	1	1
230–240	8	2	3	2	5	4	1	1	13	7	2	1
240–250	9	4	4	2	4	3	1	0	15	6	1	1
250–260	4	10	3	7	4	7	1	4	14	7	5	1
260–270	10	4	1	1	5	4	3	3	16	8	4	3
270–280	9	2	3	1	8	6	1	2	11	8	5	3
280–290	12	8	7	7	7	8	4	6	18	5	10	4
290–300	6	5	9	3	4	8	6	2	13	13	6	5
300–310	5	5	6	1	4	9	4	6	10	17	4	4
310–320	8	4	5	4	5	8	4	4	14	5	6	1
320–330	8	5	6	7	5	0	2	3	12	8	9	0
330–340	3	7	3	7	8	7	4	4	9	12	3	0
340–350	7	3	3	1	5	1	2	1	10	3	3	2
350–360	5	2	4	2	3	4	2	2	11	6	3	0

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every* 10° *of Direction*—continued.

Angle of Direction.	6^h–7^h.				7^h–8^h.				8^h–9^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	9	5	2	0	5	5	2	1	6	2	2	0
10–20	9	1	2	0	3	4	2	0	2	1	1	0
20–30	7	1	1	0	8	2	0	0	4	1	0	0
30–40	4	5	2	0	2	0	0	0	0	0	0	0
40–50	9	1	0	0	6	0	0	0	1	0	0	1
50–60	5	1	0	0	3	0	0	1	2	0	0	0
60–70	9	0	0	0	4	0	0	0	2	0	0	0
70–80	5	0	1	0	2	0	0	0	3	1	1	0
80–90	1	2	1	0	2	0	0	0	1	0	0	0
90–100	1	0	0	0	4	2	0	1	1	3	0	0
100–110	5	2	0	0	1	0	0	0	0	1	0	0
110–120	4	1	1	0	3	0	0	0	0	0	0	0
120–130	0	1	0	0	1	0	0	0	4	1	0	0
130–140	5	2	0	0	3	1	1	0	2	0	0	0
140–150	7	2	1	0	2	1	0	0	4	0	0	0
150–160	8	2	0	1	1	3	0	0	2	0	1	0
160–170	14	3	0	2	1	2	0	0	1	2	0	1
170–180	7	1	2	0	9	1	1	0	2	3	1	1
180–190	3	4	4	0	6	3	2	2	1	4	0	1
190–200	8	2	4	0	4	6	3	2	5	5	2	2
200–210	4	6	1	3	9	0	4	1	4	8	2	6
210–220	6	0	1	2	9	5	0	2	7	6	4	1
220–230	19	10	2	0	12	8	7	0	8	3	6	0
230–240	8	12	3	0	8	7	1	2	10	6	4	2
240–250	13	6	5	3	18	8	8	6	9	10	5	3
250–260	12	9	6	3	14	7	7	4	12	9	6	5
260–270	22	9	9	4	14	5	6	4	15	8	5	4
270–280	20	13	6	4	6	8	1	4	8	11	3	2
280–290	20	12	3	5	12	13	3	2	6	9	2	4
290–300	19	18	6	1	8	11	4	7	8	7	2	4
300–310	14	20	4	6	8	5	7	2	4	8	3	4
310–320	21	12	8	3	15	6	6	1	5	5	8	0
320–330	19	12	6	0	11	12	1	3	7	4	1	1
330–340	16	13	4	3	11	10	6	0	7	4	3	0
340–350	15	11	4	0	8	4	2	1	4	8	1	0
350–360	16	7	3	0	4	5	3	0	5	4	2	0

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	9^h–10^h.				10^h–11^h.				11^h–12^h.			
	0″·2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″·2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″·2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
° ° 0–10	5	2	0	1	3	0	0	0	4	0	1	0
10–20	0	1	0	0	1	0	0	0	1	3	0	0
20–30	3	2	0	0	2	2	0	0	0	1	0	0
30–40	5	1	0	0	0	1	0	0	2	0	0	0
40–50	1	1	0	0	0	0	1	0	3	2	0	0
50–60	0	0	0	0	0	1	0	0	1	1	0	2
60–70	2	0	0	0	1	0	0	0	3	0	1	0
70–80	0	0	0	0	3	0	0	0	1	1	0	0
80–90	6	0	0	0	1	2	0	0	2	1	0	1
90–100	5	0	1	0	3	1	0	0	1	0	0	0
100–110	2	0	0	1	0	0	0	0	0	0	0	0
110–120	2	2	0	0	0	1	0	0	1	1	1	0
120–130	1	3	0	2	1	0	0	0	1	0	0	1
130–140	1	2	0	1	2	0	0	0	3	1	1	0
140–150	5	0	0	0	2	4	0	1	4	2	1	1
150–160	6	1	1	0	4	0	1	1	2	0	4	3
160–170	3	3	0	0	3	1	4	3	2	2	1	4
170–180	5	1	2	0	3	5	6	4	5	6	9	6
180–190	5	4	4	2	4	4	2	7	2	2	6	7
190–200	9	4	5	7	4	6	6	0	4	9	6	6
200–210	5	6	4	2	2	5	8	11	7	6	7	5
210–220	5	4	3	5	2	5	4	6	4	4	2	3
220–230	6	11	2	2	3	2	7	3	2	6	2	1
230–240	5	9	4	2	10	7	3	3	5	7	1	1
240–250	6	4	2	1	6	2	3	1	1	2	3	3
250–260	7	2	1	3	6	2	4	0	4	3	1	2
260–270	11	7	4	3	6	5	4	3	8	2	2	3
270–280	6	9	2	3	6	1	6	3	2	2	0	1
280–290	9	8	2	2	9	2	3	2	4	4	1	3
290–300	5	4	6	4	1	6	1	3	3	3	0	1
300–310	4	4	1	1	9	4	5	2	2	2	0	1
310–320	4	4	2	0	3	1	4	3	4	0	2	2
320–330	5	3	5	0	2	7	3	2	3	3	3	0
330–340	1	3	3	0	3	5	3	3	7	1	0	4
340–350	5	3	2	0	1	4	4	2	2	2	3	1
350–360	6	4	0	0	2	1	0	0	2	0	0	2

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	12h–13h.				13h–14h.				14h–15h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
°–°												
0–10	3	1	1	0	4	1	1	1	5	0	1	1
10–20	3	1	0	0	0	0	0	0	6	3	0	0
20–30	3	1	0	1	2	3	0	0	5	1	1	0
30–40	2	0	0	0	2	1	1	0	2	1	2	2
40–50	2	2	0	0	1	0	1	0	1	2	0	0
50–60	3	1	0	0	1	0	2	0	1	0	2	0
60–70	2	0	0	0	3	2	0	0	1	0	0	0
70–80	1	0	1	0	0	1	1	0	2	0	0	1
80–90	2	0	1	0	4	1	1	0	3	3	1	0
90–100	2	1	1	0	3	0	0	0	5	1	3	0
100–110	0	0	2	0	1	0	0	0	2	1	1	0
110–120	0	1	0	0	2	1	1	1	1	3	0	0
120–130	4	2	0	0	1	1	0	1	5	4	1	0
130–140	2	2	0	1	4	3	1	1	4	2	3	1
140–150	2	1	3	1	3	2	3	4	1	1	4	3
150–160	1	3	0	1	4	1	4	5	4	4	2	6
160–170	6	7	3	4	6	3	2	4	4	1	2	5
170–180	5	3	6	7	6	4	7	7	5	4	10	2
180–190	6	5	4	1	4	6	7	5	4	2	3	2
190–200	10	4	2	4	4	3	4	4	3	4	8	4
200–210	3	3	6	5	5	5	0	2	2	4	0	2
210–220	9	4	6	5	1	1	1	2	7	1	2	4
220–230	4	5	2	3	3	4	7	3	1	2	3	2
230–240	5	5	3	2	4	2	4	3	1	3	3	2
240–250	7	4	2	2	3	6	3	2	4	1	2	1
250–260	2	3	1	3	3	4	4	3	4	1	1	0
260–270	2	3	3	1	4	2	1	0	9	2	1	2
270–280	4	0	1	3	3	2	0	1	5	3	1	3
280–290	2	4	3	3	3	2	0	3	3	3	1	1
290–300	4	7	2	5	1	2	1	3	3	3	1	1
300–310	1	2	3	4	2	4	2	1	1	4	1	1
310–320	2	5	4	2	3	3	3	0	4	1	4	0
320–330	5	3	2	5	3	2	4	0	4	2	3	1
330–340	3	2	2	1	4	1	3	2	6	1	4	0
340–350	2	2	1	2	2	3	1	0	2	6	1	0
350–360	6	5	2	1	1	2	1	0	7	1	2	1

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	15^h–16^h.				16^h–17^h.				17^h–18^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
° °												
0–10	3	0	1	1	6	5	0	0	8	2	1	0
10–20	2	2	0	0	6	2	1	0	3	2	0	0
20–30	2	1	0	0	6	2	1	0	5	2	0	1
30–40	2	2	0	0	2	0	1	0	1	2	0	0
40–50	3	2	0	2	1	0	0	0	3	3	0	0
50–60	1	1	0	0	3	0	0	0	3	6	1	0
60–70	3	1	0	1	3	0	0	0	3	2	2	1
70–80	2	1	0	0	2	0	0	1	3	1	3	0
80–90	2	0	0	0	3	0	0	1	4	1	1	0
90–100	1	1	0	0	5	1	1	1	4	3	3	3
100–110	6	2	0	0	3	4	0	0	6	1	0	2
110–120	2	2	1	1	2	1	0	2	8	1	2	0
120–130	5	1	0	2	5	3	1	2	4	1	1	2
130–140	3	6	7	1	2	1	2	2	5	5	1	1
140–150	4	3	3	4	5	0	1	1	6	2	1	0
150–160	8	3	3	4	0	3	2	0	11	2	1	1
160–170	6	3	2	3	4	4	0	2	5	0	1	1
170–180	4	3	2	0	3	3	3	3	6	4	0	1
180–190	5	1	1	2	2	3	0	1	7	4	1	2
190–200	6	2	4	2	3	2	1	1	6	2	1	1
200–210	3	3	3	4	5	3	0	1	7	4	3	0
210–220	6	5	2	3	6	0	2	0	9	1	2	0
220–230	5	3	1	3	7	4	2	2	12	8	4	1
230–240	7	2	0	3	6	4	4	2	5	5	6	1
240–250	2	2	2	5	6	6	2	4	11	7	2	4
250–260	8	2	1	0	13	4	4	3	11	4	2	3
260–270	6	8	2	1	6	6	4	2	17	4	8	3
270–280	7	4	4	2	6	10	3	2	8	5	3	2
280–290	6	2	3	2	4	5	5	1	9	5	6	4
290–300	3	6	0	3	6	5	1	0	8	9	7	1
300–310	6	0	1	4	8	1	4	1	5	5	5	3
310–320	4	3	1	3	6	2	5	1	10	8	5	0
320–330	5	1	3	3	3	1	3	1	12	10	1	0
330–340	2	3	1	1	9	2	7	1	8	5	6	0
340–350	4	2	2	0	4	6	2	1	7	10	3	1
350–360	2	4	3	0	8	3	3	0	4	2	3	1

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	18h–19h.				19h–20h.				20h–21h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
°0–10°	9	11	2	0	17	15	4	2	21	10	5	1
10–20	12	3	2	1	25	17	5	1	14	11	6	3
20–30	18	12	4	0	24	20	3	1	20	12	5	2
30–40	13	6	3	1	21	15	4	4	11	7	6	0
40–50	12	3	0	2	19	7	7	4	15	10	3	0
50–60	7	4	2	3	18	6	2	1	17	11	3	1
60–70	9	4	3	2	17	3	1	3	12	9	0	5
70–80	5	3	1	2	12	5	2	0	6	3	2	0
80–90	6	4	2	0	18	3	1	1	13	5	0	0
90–100	5	3	1	1	15	4	1	1	12	7	1	0
100–110	8	1	1	1	8	2	0	0	8	2	0	0
110–120	9	5	1	0	11	2	0	0	2	1	0	0
120–130	8	2	0	0	15	2	2	0	13	4	0	0
130–140	8	2	1	0	9	1	0	0	15	4	2	0
140–150	5	4	0	1	9	3	1	0	8	2	0	0
150–160	12	3	1	0	9	3	0	0	7	3	2	0
160–170	7	4	1	1	10	2	0	0	5	2	1	1
170–180	10	3	1	0	10	2	0	1	8	6	2	0
180–190	7	4	0	0	11	3	1	0	10	7	2	0
190–200	8	5	1	0	15	4	0	0	10	5	0	0
200–210	8	4	1	0	10	3	3	1	10	7	1	1
210–220	12	3	0	0	13	4	2	0	20	2	0	2
220–230	10	5	2	2	11	3	2	3	15	9	3	1
230–240	6	6	4	0	14	5	4	2	9	7	3	3
240–250	13	2	0	2	13	4	7	0	13	5	3	2
250–260	6	4	0	4	10	7	2	6	13	4	4	2
260–270	14	4	6	3	26	3	3	3	20	7	7	1
270–280	14	4	2	2	20	6	5	0	18	4	2	1
280–290	10	8	5	3	20	6	1	2	12	7	2	2
290–300	17	10	9	2	23	8	0	2	14	5	2	1
300–310	10	8	3	2	15	9	3	2	15	7	2	0
310–320	18	8	8	0	24	9	4	0	19	8	1	0
320–330	8	6	5	0	24	17	5	4	13	8	0	1
330–340	17	5	0	2	19	11	4	0	19	7	2	1
340–350	15	8	4	3	23	12	6	2	19	7	5	0
350–360	12	11	3	1	23	13	5	0	18	12	2	3

TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	21^h–22^h.				22^h–23^h.				23^h–0^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	13	23	7	7	10	8	7	9	12	14	6	6
10–20	10	14	5	5	10	7	9	3	4	8	5	11
20–30	11	3	7	3	10	6	3	5	11	3	3	3
30–40	10	12	3	2	11	6	3	0	11	4	2	1
40–50	16	7	0	3	6	6	1	1	7	3	1	0
50–60	14	5	4	0	8	2	0	0	1	1	0	0
60–70	10	0	1	0	9	1	0	0	2	2	1	0
70–80	9	3	1	0	5	0	2	0	6	4	1	0
80–90	8	4	2	0	5	2	0	1	5	1	0	0
90–100	13	2	0	0	3	1	0	0	6	1	0	1
100–110	6	3	0	1	6	1	0	1	5	2	0	0
110–120	3	3	0	0	6	2	1	1	1	1	1	0
120–130	7	3	0	0	8	1	0	0	1	2	0	0
130–140	8	0	0	0	5	3	0	0	7	3	0	0
140–150	4	1	1	1	4	2	2	0	3	2	1	1
150–160	5	1	1	0	7	4	0	0	10	3	0	0
160–170	8	1	1	0	9	4	3	1	3	2	1	0
170–180	12	2	5	0	15	4	2	3	7	3	1	0
180–190	8	5	3	0	9	5	4	1	5	5	2	1
190–200	7	3	1	0	12	2	5	0	7	5	5	0
200–210	11	7	2	0	14	3	2	5	9	5	3	0
210–220	13	3	2	4	11	7	4	2	9	4	3	2
220–230	10	9	3	2	4	3	2	2	4	5	4	1
230–240	13	6	3	4	11	6	3	5	4	4	1	2
240–250	16	5	2	0	9	3	1	6	8	4	3	2
250–260	7	4	1	2	10	5	1	2	6	3	1	2
260–270	10	4	4	1	4	4	5	3	3	5	2	2
270–280	7	3	0	1	5	3	0	1	7	2	2	0
280–290	10	7	2	2	5	5	0	1	5	2	4	1
290–300	8	2	1	0	7	0	3	2	7	1	2	0
300–310	5	1	1	1	7	1	0	1	7	6	2	1
310–320	6	6	3	0	4	5	1	1	10	2	1	2
320–330	10	2	4	0	12	1	1	1	10	5	0	5
330–340	7	9	1	0	16	6	1	0	10	5	3	1
340–350	15	6	2	4	12	6	1	1	8	5	5	4
350–360	13	8	5	7	11	11	7	2	10	9	5	8

The initial difficulty of deciding what is the best material to work with is caused by this factor of accidental errors. To adopt arbitrary limits is not free from objection, but to take in everything is to swamp the best with inferior material. For this and other reasons the difficulty has been met by breaking up the large amount of material available into four categories. Proper motions within the limits of 0″ to 3″ per century are not used at all, and the rest are divided into sections 3″ to 5″, 5″ to 8″, and 8″ and upwards, the statistics being exhibited separately.

The figures of Table I. were plotted on various diagrams, of which that given in Plate 8 may be taken as a specimen. This gives the number of proper motions (1) from 3″ to 5″, (2) from 5″ to 8″, and (3) >8″. Similar diagrams were made, using all the proper motions >5″. The observations were smoothed by combining three consecutive azimuths, so that the ordinate at 5° is the number of stars between 350° and 20°. Further smoothing was effected by combining three hours together, giving a double weight to the middle one. Various other diagrams were made with less smoothing.

The existence of two star-streams was shown clearly, but no sufficient evidence of Stream III. was obtained.

Stream I. was very marked, especially the high peaks near 0^h and 12^h. The position of the apex was found to be at R.A. 6^h, Dec. $-8°$, having regard to the directions alone, but the relative numbers of stars about 6^h and 18^h which belonged to the stream indicated a more southerly declination, and no certain correction was indicated to the value $-13°$, the mean of the values in Professor Eddington's *Stellar Motions and the Structure of the Universe*, p. 125.

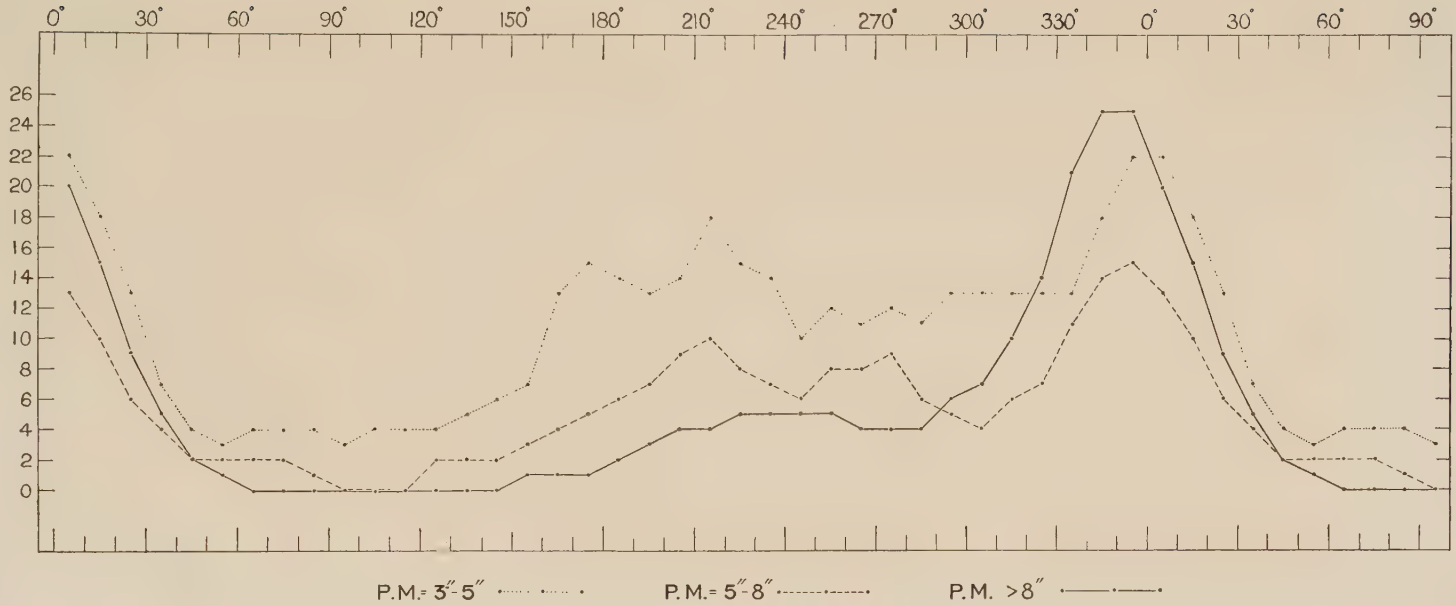
The position of the apex of Stream II. cannot be determined so accurately as that of Stream I., and the mean of several different determinations is R.A. 18^h, Dec. $-60°$.

The ratio of the stream velocities in Streams I. and II. was determined graphically by comparison of the heights of the peaks in the various diagrams. At the distance θ_1 from the apex of Stream I. the stream velocity is $V_1 \sin \theta_1$, and at a distance θ_2 from the apex of Stream II. the stream velocity is $V_2 \sin \theta_2$. For the whole region from 12^h to 0^h $\sin \theta_2$ is very nearly 1 throughout this zone. It was therefore necessary to find in what parts of the zone the peaks due to Stream I. rose to the mean height of those due to Stream II. from 12^h to 0^h. This was found to be when $\sin \theta_1 = \cdot 60$, and thus V_2 was roughly found $= \cdot 6 V_1$.

The numbers of the stars belonging to the two streams appear to be in the ratio of about 4 : 3. The ratio found by Professor Eddington from the stars of Boss's Catalogue is 3 : 2.

The constants of the two star-streams as determined from a narrow zone naturally do not possess any great accuracy. The larger proper motions give the best agreement, the smaller ones showing deviations which may arise from local irregularities in the movements of the stars or from errors in the proper motions arising from small systematic errors in the observations.

DISTRIBUTION OF PROPER MOTIONS IN DIRECTION R.A. 0^H-1^H



But in the main, as far as this discussion goes, the general features of the proper motions of the stars in this zone may be said to be well explained by the existence of two star-streams.

Part II.

The Distribution of Proper Motions in Relation to the Galaxy.

The number of stars with proper motions between varying limits were counted in each hour. They are given in detail in Table II. and in a more condensed form in Table III.

Table II.

Number of Stars for varying ranges of Centennial Proper Motion arranged in order of Right Ascension.

Limits of R.A.	Centennial Proper Motion.									
	0″·0–0″·9.	1″·0–1″·9.	2″·0–2″·9.	3″·0–3″·9.	4″·0–4″·9.	5″·0–5″·9.	6″·0–6″·9.	7″·0–7″·9.	8″·0–8″·9.	9″·0–9″·9
h h										
0–1	26	68	107	82	56	31	20	21	12	12
1–2	34	68	81	65	54	34	18	12	11	10
2–3	26	59	61	64	43	30	26	21	10	6
3–4	33	62	73	74	58	40	19	12	10	11
4–5	19	51	56	67	40	17	17	14	9	6
5–6	62	117	136	110	74	41	23	18	6	4
6–7	59	126	186	124	74	49	32	12	12	8
7–8	39	102	91	82	63	36	27	14	7	5
8–9	27	61	75	79	57	33	22	11	7	7
9–10	23	52	78	64	49	25	18	14	6	9
10–11	12	31	52	50	38	35	21	26	10	13
11–12	13	37	43	52	29	24	22	12	13	12
12–13	18	43	60	59	34	25	22	19	12	9
13–14	18	38	42	38	39	25	27	19	9	8
14–15	23	52	57	46	30	31	27	14	16	10
15–16	20	64	60	50	39	19	20	13	10	11
16–17	31	59	79	56	43	34	14	17	5	7
17–18	40	79	125	82	60	49	22	13	7	7
18–19	58	164	145	119	61	31	33	14	14	6
19–20	120	223	228	154	87	43	30	18	13	11
20–21	88	187	194	148	81	37	19	22	10	1
21–22	62	123	158	99	80	41	25	11	5	5
22–23	57	98	132	85	48	39	22	14	13	12
23–24	48	83	100	84	56	31	22	17	8	5
Sums	957	2047	2419	1933	1293	800	548	378	235	195
Means	40	95	101	81	54	33	23	16	10	8

TABLE II.—*Number of Stars for varying ranges of Centennial Proper Motion arranged in order of Right Ascension*—continued.

Limits of R.A.	Centennial Proper Motion.								
	10″·0–10″·9.	11″·0–11″·9.	12″·0–12″·9.	13″·0–13″·9.	14″·0–14″·9.	15″·0–19″·9.	20″·0–29″·9.	30″·0–49″·9.	50″·0+
h h									
0–1	10	6	4	5	7	9	7	4	0
1–2	8	6	3	5	1	10	7	4	2
2–3	7	4	2	4	3	10	3	4	2
3–4	9	1	0	4	4	4	3	3	1
4–5	6	8	3	2	1	3	5	2	0
5–6	5	4	0	4	0	4	4	1	0
6–7	4	3	1	1	3	4	2	4	1
7–8	7	5	4	2	3	5	4	3	3
8–9	5	5	4	4	1	6	3	4	2
9–10	5	6	6	1	3	5	6	2	1
10–11	12	6	4	2	3	12	9	4	0
11–12	6	6	3	0	7	5	4	3	1
12–13	5	5	7	6	3	8	12	1	0
13–14	5	7	8	2	3	10	4	5	3
14–15	4	5	2	1	2	10	5	2	2
15–16	11	3	4	6	5	9	3	0	2
16–17	11	2	2	3	0	4	3	0	3
17–18	2	3	5	5	2	4	2	4	2
18–19	4	3	1	3	4	3	2	2	1
19–20	3	1	3	3	0	8	7	2	2
20–21	4	4	1	3	2	2	4	2	0
21–22	9	3	1	6	6	6	6	3	1
22–23	5	7	5	3	2	6	8	1	0
23–24	8	3	4	3	6	10	6	3	5
Sums	155	106	77	78	71	157	119	63	34
Means	7	4	3	3	3	7	5	$2\frac{1}{2}$	$1\frac{1}{2}$

DISTRIBUTION OF PROPER MOTIONS

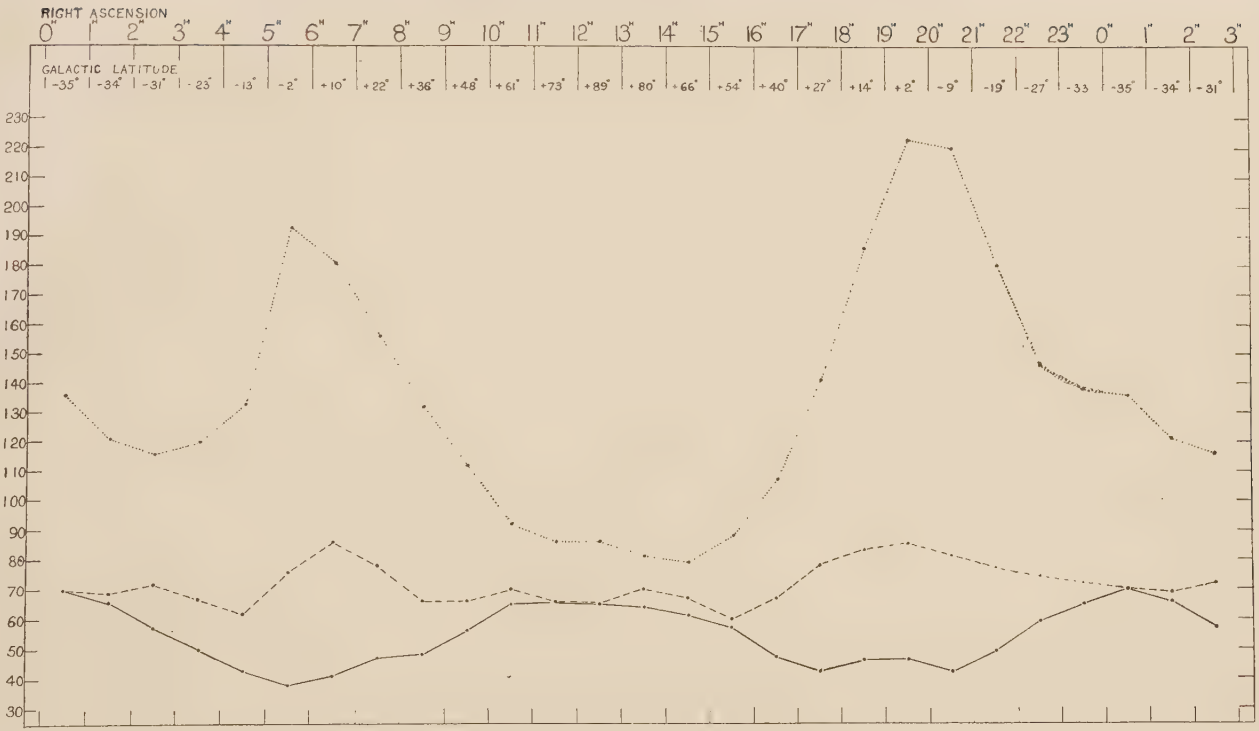


TABLE III.

Numbers and Percentages for varying Groups of Centennial Proper Motion with Galactic Latitude for each hour of R.A.

Limits of R.A.	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	Total No. Stars.	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	Galactic Latitude.
h h	Numbers.					Percentages.				
0–1	201	135	72	76	487	41	28	15	16	−35°
1–2	183	119	64	67	433	42	27	15	16	34
2–3	146	107	77	55	385	38	28	20	14	31
3–4	168	133	71	50	422	40	31	17	12	23
4–5	126	107	48	45	326	39	33	15	14	13
5–6	315	184	82	32	613	51	30	13	6	− 2
6–7	371	198	93	43	705	53	28	13	6	+10
7–8	232	145	77	48	502	46	29	15	10	22
8–9	163	136	66	48	413	39	33	16	12	36
9–10	153	113	57	50	373	41	30	15	14	48
10–11	95	88	82	75	340	28	26	24	22	61
11–12	93	81	58	60	292	32	28	20	20	73
12–13	121	93	76	68	358	34	26	21	19	89
13–14	98	77	71	64	310	32	25	23	20	80
14–15	132	76	72	59	339	39	22	21	18	66
15–16	144	89	52	64	349	41	26	15	18	54
16–17	169	99	65	40	373	45	27	17	11	40
17–18	244	142	84	43	513	48	27	16	9	27
18–19	367	180	78	43	668	55	27	12	6	14
19–20	571	241	91	53	956	60	25	10	5	+ 2
20–21	469	229	78	33	809	58	28	10	4	− 9
21–22	343	179	77	51	650	53	27	12	8	19
22–23	287	133	75	62	557	52	24	13	11	27
23–24	231	140	70	71	512	45	27	14	14	−33

Some of the features of Table III. are shown in the accompanying diagram (Plate 9). It will be seen that the number of stars with proper motions greater than 8″ a century is a maximum about 0^h and 12^h and a minimum about 6^h and 18^h. The minimum at 18^h is not so low as that at 6^h. This distribution seems to be in general satisfactorily explained by reference to the positions of the apices of the two star-streams. Taking the apex of Stream I. at R.A. 6^h Dec. −13°, its distance from the zone is nearly 90° at 0^h and 12^h, 40° at 6^h, and 165° at 18^h. This would give maxima at 0^h and 12^h, minima at 6^h and 18^h, that at 6^h being somewhat lower. Taking the apex of Stream II.—which is, however, the slower of the two streams—at R.A. 18^h: Dec. −60°, the zone is approximately 90° distant for the whole extent from 12^h to 0^h, and 148°

at 6^h, leaving only a small stream velocity to be compounded with accidental velocities. The distribution of the stars with proper motions greater than 8″ a century appears therefore to be such as would arise from a uniform distribution of the stars in space.

The next group with proper motions 5″ to 8″ a century shows clearly defined but small maxima in the neighbourhood of the Milky Way, the numbers per hour rising from 68 to 84. Combined with the group 8″+ a century, there would be a nearly uniform distribution of 125 stars per hour.

In the two remaining groups of stars with proper motions 3″ to 5″ and 0″ to 3″, the condensation of stars as the Galaxy is approached is very marked, and the extent to which this is shared by the groups 3″ to 5″ is specially to be noted.

The question whether stars of sensible proper motion crowd towards the Milky Way is discussed by Newcomb,* who makes the following remarks:—"Kapteyn has examined this question in the case of the Bradley stars. His conclusion is that those having a considerable proper motion, say more than 5″ per century, are nearly equally distributed over the sky, but that when we include those having a smaller proper motion, we see a continually increasing tendency to crowd towards the galactic plane. It seems to the writer that the uncertainty as to the smaller proper motions of the Bradley stars renders this result quite unreliable. To reach a more definite conclusion, we must base our work on lists of proper motions which are as nearly complete within their limits as it is possible to make them."

Newcomb then considers the proper motions found in the Zone Catalogues of Auwers, 15° to 20° N. Dec., and Boss 1° to 5° N. Dec., and concludes that no crowding is shown. The proper motions dealt with by Auwers and Boss are in the main large ones, and the present investigation suggests that Newcomb's results are to be attributed to this cause.

The increasing tendency to crowd towards the Milky Way, starting with proper motions between 5″ and 8″ a century, and shown very strongly in stars with proper motions 3″ to 5″ a century, suggested more detailed investigation.

The numbers of Tables II. and III. were accordingly rearranged according to their galactic latitude given in the last column of Table III. The means were then taken for five separate groups, giving the following table:—

Limits of Gal. Lat.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15″.
± 2° to ±10°	82	163	186	134	79	42	26	17	20	8	9
±13 to ±27	44	96	111	87	59	36	24	13	23	14	12
±31 to ±36	32	68	85	75	53	32	22	17	25	16	20
±40 to ±61	22	52	67	55	42	28	18	17	27	12	16
±66 to ±89	18	42	50	49	33	26	25	16	27	17	19

* *The Stars: A Study of the Universe*, p. 252.

Dividing the numbers in each column by the number in the last row, the ratio of the densities in different galactic latitudes to that near the galactic pole is found for stars of proper motion 0″ to 1″, 1″ to 2″, etc., as follows:—

Relative Density of Stars of different Proper Motions at Different Galactic Latitudes compared with the Density near the Galactic Pole.

Galactic Latitude.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15″.
±6°	4·6	3·8	3·7	2·7	2·4	1·6	1·1	1·1	0·8	0·5	0·5
±21	2·5	2·3	2·2	1·8	1·8	1·4	1·0	0·8	0·9	0·9	0·6
±34	1·8	1·6	1·7	1·5	1·6	1·3	0·9	1·1	0·9	1·0	1·1
±51	1·3	1·3	1·3	1·1	1·3	1·1	0·7	1·1	1·0	0·7	0·8

As the first group covers the regions where the zone cuts the Milky Way, it has been separated. The first region, extending from R.A. 5^h to 7^h, contains 1318 stars; the second, extending from 19^h to 21^h, 1765 stars, and the densities relative to that near the galactic pole are as follows:—

Relative Densities at Intersection with Galactic Planes.

Limits of R.A.										
5^h–7^h	3·5	2·9	3·2	2·3	2·2	1·7	1·1	0·9	0·7	0·5	0·5
19^h–21^h	5·9	4·9	4·2	3·0	2·5	1·5	1·0	1·2	0·8	0·5	0·4

An inspection of the table shows that in the region of greater density there is a corresponding increase of condensation for stars of smaller proper motion.

An interesting comparison may be made between the area 23^h to 3^h which is at galactic latitude $-35°$ with that of 11^h to 15^h in galactic latitude $+77°$. The effect of star streaming is nearly the same in these two regions. We find—

Relative Density at the Extreme Galactic Latitudes.

	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15′.
23^h–3^h	48	93	124	88	60	36	22	16	26	18	19
11^h–15^h	18	42	50	49	33	26	25	16	27	17	19
Difference	30	51	74	39	27	10	−3	0	−1	+1	0
Ratio	2·7	2·2	2·5	1·8	1·8	1·4	1	1	1	1	1

Thus the two regions agree closely in the numbers of stars with proper motions >6″ a century, but the ratio continually increases as the Galaxy is approached, more and more as the amount of proper motion diminishes. Although the numbers in this table and previous tables are affected considerably by the accidental errors of the proper motions, the ratios are affected to a smaller extent. The effect has probably been to increase the ratios somewhat between 4″ to 6″ and diminish them between 0″ to 2″. The differences in the last table may be compared with the distribution

which would arise if these stars had no proper motions at all, and the spreading was due to accidental errors alone.

Limits of P.M.	Assumed Probable Error.			Observed.
	1″·7.	1″·5.	1″·3.	
″ ″ 0–1	7·5	9·6	12·7	13·2
1–2	19·5	23·7	29·0	22·4
2–3	23·9	26·4	28·6	32·5
3–4	20·8	20·4	18·2	17·1
4–5	14·9	11·9	8·1	11·8
5–6	8·1	5·4	2·6	3·1
6–7	3·7	1·9	0·7	...
>7	2·1	0·7	0·1	...

These figures show that the probable error is between 1″·3 and 1″·5, agreeing with the statement at the beginning of the paper. It is, however, difficult to free the actual numbers of these tables from the effect of accidental error in the determination of proper motion.

The most natural explanation of these variations is to be found in the way in which star density varies in space. In the region nearest to us the density is a maximum, and diminishes as we proceed outwards, but much more rapidly in the direction of the galactic pole, so that the surfaces of equal density are spheroids of continually increasing oblateness. Stars with proper motions greater than 6″ a century in the main lie in the near region where the surfaces of equal density are spheres. But stars with proper motions of 3″ a century are in the mean at distances where the density is much greater in the direction of the plane than in that of the pole of the Galaxy.

Further Notes on the Radiative Equilibrium of the Stars. By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor. (Plate 10.)

13. *Ionisation in the Stars.*—In the first paper * calculations were made on the assumption of an average molecular weight 54 for the material of a star. In the present paper the average molecular weight will be taken to be 2.

The first value fairly represents the hypothesis that the ultimate particles, which move independently, are atoms. It will be shown that the value 2 represents the hypothesis that the atoms are highly ionised, so that most or all of the electrons outside the nucleus have been broken off and move as independent particles. The suggestion that at these high temperatures we are concerned

* P. 16. The paragraphs and formulæ of the present paper are numbered in continuation of those in the first paper.

with particles smaller than the atom was made to me independently by Newall, Jeans, and Lindemann. By an argument which now appears insufficient, I had supposed that atomic disintegration, though undoubtedly occurring, could not have proceeded very far; but Jeans has convinced me that a rather extreme state of disintegration is possible, and indeed seems more plausible. In any case, the prevalence of this view of the interior state of a star makes it desirable to present the theory of stellar temperatures and densities on this alternative basis.

The principal result of the first paper—that the effective temperature of a giant star varies as the sixth root of the density—is not changed by taking account of ionisation. The main difference in the results is that the influence of the star's *mass* on the temperature is considerably increased by ionisation. The change seems on the whole to improve the agreement with observation in minor details; but since the theory is necessarily only a rough approximation, it is not desirable to lay too much stress on this removal of small discrepancies. A more important confirmation is that the theory, when thus modified by introducing ionisation, provides an explanation of why stellar masses are of the order of magnitude actually found.

It will be remembered that the temperatures within the stars are chiefly from 10^6 to 10^7 degrees. The radiation at these temperatures consists mainly of waves a little longer than X-rays, having strong ionising power. But since we do not know how fast recombination of the ions takes place, it is difficult to predict what proportion of the atoms are ionised at any moment. For many elements the first electron is removed by an ionisation potential of about 4 volts; and, in accordance with the theory of the photoelectric effect, this separation can be effected by radiation of wave-length 3200 Å or less (since a quantum of this frequency carries sufficient energy). At 9000° the maximum radiation is of the required wave-length. Although the theory gives no certain guidance, it seems reasonable to suppose that when the material is in equilibrium with radiation at temperature 9000° a fair proportion of the atoms must be in the ionised state, and that the proportion will increase rapidly as the temperature rises. We should expect then the loss of one electron to take place at temperatures of the order 10,000°; succeeding electrons will be lost at higher temperatures which can scarcely be predicted; but we can estimate on the same basis the temperature at which the last electron is removed, since the nucleus attended by a single electron forms the simple system considered by Bohr. The necessary frequency is given by the formula $2\pi^2 N^2 me^4/h^3$, where N is the atomic number, m and e the mass and charge of a negative electron, and h Planck's constant. The wave-length is, in fact, $\frac{3}{4}$ that of one of the characteristic X-rays of the element. For aluminium it is 5·4 Å; for copper 1·1 Å. The maximum radiation at 10^7 degrees is of wave-length 2·9 Å. It seems probable therefore that, though disintegration is nearing completion, it will

scarcely be complete even in the hottest part of the stars, and that all except the lightest elements will retain a small remnant of their electrons.

If N electrons break away from each atom, they will act like independent particles in causing pressure, so that for the same temperature and density the pressure is increased $(N+1)$fold. Using the term molecules to denote the ultimate particles, whether atoms or electrons, the average molecular weight will be $A/(N+1)$, where A is the atomic weight; and it is this average which occurs as the molecular weight in the pressure-temperature-density equation of a gas. (I assume that the electric charges of the particles do not affect the calculation of the pressure.) Now the atomic number, or number of electrons outside the nucleus, is very near to half the atomic weight for all elements except hydrogen. Hence, if disintegration is complete, $A/(N+1)$ is approximately equal to 2, whatever may be the composition of the star. Although 2 is thus the extreme lower limit for the molecular weight, the retention of the last few electrons would make comparatively little difference; thus iron (atomic number 26) might retain 8 electrons and gold (79) might retain 14 without the molecular weight increasing above 3.

It may be suggested that the nucleus could also be broken up, leading to a further diminution of the average molecular weight. But it seems probable that this would require much greater energy; and it could scarcely be accomplished by radiation of less frequency than the characteristic X-rays.

14. *Summary of the Theoretical Formulæ.*—The theory given in the first paper will here be collected into a form more convenient for reference and development. I wish especially to show which formulæ are general, and which depend on particular assumptions.

The notation may be summarised as follows:—

ξ is the distance from the centre of the star.
p, ρ, T, the pressure, density, and temperature of the material.
k is the mass-coefficient of absorption.
$\frac{4}{3}\pi B$ is the net outward flow of radiation per unit area.
a is the constant $7{\cdot}06 \times 10^{-15}$ such that aT^4 is the density of the radiant energy.
μ is a constant equal to $ac/4\pi$ where c is the velocity of light.*

The two fundamental equations are

$$d(p + \tfrac{4}{3}aT^4) = -g\rho d\xi \quad . \quad . \quad . \quad (34)$$

$$d(T^4) = -\frac{B}{\mu} \cdot k\rho d\xi \quad . \quad . \quad . \quad (35)$$

Equation (34) is the usual hydrostatic equation with the correction for radiation-pressure, $d(\frac{4}{3}aT^4)$, as found in (24). Equation (35) is the equation of radiative equilibrium, expressing that the flow of energy, B, is proportional to the gradient of the energy-density,

* This value of μ was not given previously, but can easily be verified.

$d\mathrm{T}^4/d\xi$, and inversely proportional to the opacity of the medium, $k\rho$; it is equivalent to (6). Both equations are perfectly general, and hold for any kind of material; they could only fail if some additional forces or effects were present, *e.g.* electric forces or convection currents.

Eliminating $d\xi$, we have,

$$d(p + \tfrac{4}{3}a\mathrm{T}^4) = \frac{g\mu}{\mathrm{B}} \cdot \frac{1}{k} d(\mathrm{T}^4) \quad . \quad . \quad . \quad (36)$$

The surface-value of B is given by the condition that the net outward flow of radiation near the boundary is $\pi\mu\mathrm{T}_1^4$ per unit area, T_1 being the effective temperature.* It follows that the surface-value of $\frac{g\mu}{\mathrm{B}}$ is $\frac{4}{3}\frac{g_1}{\mathrm{T}_1^4}$ where g_1 is the value of g at the surface. Let the ratio B/g at any interior point be η times the surface-value, then

$$d(p + \tfrac{4}{3}a\mathrm{T}^4) = \tfrac{4}{3}\frac{g_1}{\mathrm{T}_1^4} \cdot \frac{1}{\eta k} d(\mathrm{T}^4) \quad . \quad . \quad . \quad (37)$$

Now g is proportional to the mass within a sphere of radius ξ, divided by ξ^2; B is proportional to the energy liberated per unit time in the same sphere, divided by ξ^2. Hence η varies as the energy liberated per unit mass, averaged through the part of the star interior to the point considered. Owing to this averaging, η varies very little even when the energy is liberated at very different rates in different parts of the star. For example, if the radiated energy comes entirely from contraction, and the star passes through a series of homologous states of gradually increasing density, the energy liberated in any volume per unit time will be proportional to the energy present in that volume. In that case, with the law of temperature considered in the first paper ($p \propto \mathrm{T}^4$), I find that η rises slowly from 1 at the surface to 1·7 at the centre.

We shall henceforth introduce an approximation, setting $\eta = 1$. From (37) it appears that the effect of this will be equivalent to altering the value of k, so that the error introduced may be amalgamated with the uncertainties of k. η will almost certainly rise a little above its surface-value unity, since the greatest secular change of temperature and density is likely to take place near the centre. Hence the true coefficient of absorption will be a little less than the k used in the formulæ.

Let

$$\frac{1}{k_0} = \frac{1}{\mathrm{T}^4}\int_0^{\mathrm{T}} \frac{1}{k} d(\mathrm{T}^4) \quad . \quad . \quad . \quad . \quad (38)$$

so that k_0 is an average value of k taken in such a way as to give enormously greater weight to the higher temperatures. If k is any slowly varying function of T, k_0 will differ very little from k, because of this mode of weighting. It will be understood that k and k_0 may be functions of the temperature, since the constitution of the radiation to be absorbed varies with the temperature.

* I take as definition of effective temperature the temperature of a black-body, which gives the same total radiation per unit area.

Integrating (37), we have

$$p = \tfrac{4}{3}\left(\frac{g_1}{k_0 T_1^4} - a\right) T^4 . \quad . \quad . \quad . \quad (39)$$

One useful result follows immediately from this formula. Since p cannot be negative,

$$k_0 < \frac{g_1}{a T_1^4} \quad . \quad . \quad . \quad . \quad (40)$$

If ξ_1 is the radius of the star, $g_1 \xi_1^2$ gives the star's mass, and $a T_1^4 \xi_1^2$ gives the total radiation emitted, *i.e.* the bolometric magnitude. Hence the right-hand side of (40) can be estimated from observation. With the observational data previously used,

$$k_0 < 31.$$

This has to hold for all temperatures up to the highest interior temperature in the star. It follows that k cannot be greater than 31, except possibly for a very limited range of temperature.* We ought to allow some margin for the uncertainty of the observational data; but it seems safe to conclude that k cannot be greater than 100. This limit is interesting, since the observed absorption-coefficients of many elements for soft X-rays considerably exceed 100. The lower value found in the stars is probably due to ionisation, which renders the affected atoms non-absorbing. Laboratory experiments refer to weak streams of radiation; for very intense streams the absorption must diminish, since there will be a saturation effect.

Introduce a quantity β defined by

$$\frac{p}{\frac{4}{3}aT^4} = \frac{\beta}{1-\beta} \quad . \quad . \quad . \quad . \quad (41)$$

where β is in general variable.

We then obtain from (39)

$$T_1^4 = \frac{1}{k_0 a} g_1 (1-\beta) \quad . \quad . \quad . \quad . \quad (42)$$

Equations (41) and (42) are formally equivalent to (26*a*) and (29).

If k_0 *is constant throughout the star*, β is constant by (42), and hence $p \propto T^4$. Thus (for a perfect gas) the temperature and density distribution is of a well-known type studied by Emden. We find also from (41) that $1-\beta$ is the ratio of the radiation-pressure to gravitation, since dp represents the excess of gravitation over radiation-pressure on an element, and $d(\frac{4}{3}aT^4)$ represents the radiation-pressure. Radiation-pressure can thus be allowed for by replacing the constant of gravitation G by βG.

For a gaseous star in which $p \propto T^4$, the central temperature and

* If k is greater than 31 at any point, the pressure there decreases inwards. If η is taken into account, the true absorption coefficient is, as we have seen, still lower.

pressure are related to the mass M, mean density ρ_m, molecular weight m, and effective constant of gravitation βG, in the ratios *

$$\left.\begin{aligned} T_0 &\propto \rho_m^{\frac{1}{3}} M^{\frac{1}{3}} \beta m \\ p_0 &\propto \rho_m^{\frac{1}{3}} M^{\frac{2}{3}} \beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (43)$$

Hence (41) leads to an equation for β of the form

$$\frac{\beta}{1-\beta} = \frac{1}{m^4 M^2 \beta^3} \times \text{constant}$$

or

$$1 - \beta = y\beta^4 \quad . \quad . \quad . \quad . \quad (44)$$

where y varies as $M^2 m^4$.

This equation enables us to find β for any giant star, provided k is constant.

In default of information as to how k varies with the temperature, I make this assumption of the constancy of k in what follows. It has been seen that it leads to the law that the bolometric magnitude of a giant star is independent of the density, which seems to be verified by observation ; and it is difficult to see how this could happen unless the assumption were approximately true. The assumption appears in two aspects, (1) that k may be taken as constant throughout a star, (2) that it may be taken constant for stars of different spectral types. With regard to (1), it seems natural that we should obtain an approximate representation by replacing the varying k_0 by some kind of average value. If this is admitted, we can by observation compare the values of k_0 (*i.e.* the aforesaid average values) for different stars by (42). Take two stars of the same mass with very different densities, say type A and type M ; their total radiations will be inversely proportional to their values of k_0.† Thus if their k_0's were in the ratio 6·3 : 1, the stars would differ bolometrically by 2 magnitudes. I think this is an upper limit to the variations which the observations will allow. The observations, therefore, seem to show that k cannot vary very much.

One apparent alternative might be considered. Suppose that k_0 has a maximum value (about 30) at some rather low temperature. It follows by (39) that aT_1^4/g_1 cannot be greater than $\frac{1}{30}$. Is it possible that a star settles into such a condition that it radiates the maximum possible amount consistent with this limit? This would give $aT_1^4/g_1 = \frac{1}{30}$ for all stars. I scarcely think this could lead to a satisfactory theory ; it involves zero pressure and density at some point in the interior of the star.

15. *Radiation-pressure for low Molecular Weight.*—The ratio of radiation-pressure to gravitation, $1-\beta$, is obtained from the quartic equation (44)

$$1 - \beta = y\beta^4,$$

* Emden, *Gaskugeln*, pp. 69, 97.

† Total radiation $\propto \xi_1^2 T_1^4 \propto \frac{M}{k_0}(1-\beta)$, and by (44) β is independent of ρ and k_0.

where y varies as M^2m^4. We have given (p. 28) the value of y for $M = 1{\cdot}5 \times \odot$, $m = 54$, viz. $\log_{10} y = 5{\cdot}2980$. Hence we can find y and β for any other values.

In Table I. the results for various masses for molecular weight 2 are given, those for molecular weight 54 being added for comparison.

TABLE I.

Mass. ($\odot = 1$)	$1-\beta$. Molecular Weight 2.	$1-\beta$. Molecular Weight 54.
0·5	·036	·920
1·0	·106	·943
1·5	·174	·953
3·0	·320	·967
4·5	·409	·973

It will be seen that $1-\beta$ is now much more sensitive to a change in mass than when the molecular weight was high. Also radiation-pressure no longer counterbalances the greater part of gravitation;* its place in sustaining the equilibrium of the star has been largely taken by the pressure of the free electrons. I am not sure that there is much fundamental difference between the two ideas; we may say that the pressure is now sustained by a mixture of cathode rays and X-rays instead of by X-rays alone.

The coefficient of absorption can now be found from (42). I assumed in the first paper that a star of mass 1·5 and density 0·002 would have an effective temperature 6500°; on reviewing the evidence I believe that this is the best value to adopt. Another way of expressing the same conclusion is that a star of mass 1·5 and type G has an absolute magnitude $-0^m{\cdot}3$. The average absolute magnitudes of giant stars are known with considerable accuracy, and the main cause of uncertainty is the doubt as to the masses to which they refer.

On this basis the value of k is 5·4 C.G.S. units. It is interesting to note that the extreme change of molecular weight has not altered the order of magnitude of k; the value previously found was 29·5.

The conclusion that atomic disintegration is not complete is important in connection with this value of k. When X-rays are absorbed, the energy is expended in ionisation; but, if the material is fully ionised, a ray traversing it can find nothing to expend itself on. In that case k should approximate to the coefficient of scattering, 0·2 (see § 12); but k is certainly higher than that.

By (43) the central temperature is modified from the value previously given proportionately to βm, or by (44) proportionately to $\sqrt[4]{(1-\beta)}$. The change is comparatively trifling.

* The importance of radiation-pressure, however, increases rapidly if the ionisation is not quite complete; *e.g.* for molecular weight 2·8 we should take from Table I. the value of $1-\beta$, corresponding to double the actual mass of the star (since $1-\beta$ depends on M^2m^4).

16. *Limit to the Masses of Stars.*—The equation (44) for determining β involves no constants derived from astronomical observation, and can be applied to theoretical gas-spheres of any size. It is therefore a striking coincidence that the actual masses of stars correspond to the most critical range of values of β, for which radiation-pressure increases from negligible to predominant influence.

To show the character of this coincidence, consider a series of gas-spheres of all sizes—from bubbles to universes.

As the mass increases from 0 to $1{\cdot}8 \,.\, 10^{33}$ gm., the ratio of radiation-pressure to gravitation in the gas-sphere increases from 0 to 0·1.

As the mass increases from $1{\cdot}8 \,.\, 10^{33}$ to $3{\cdot}7 \,.\, 10^{34}$ gm., the ratio of radiation-pressure to gravitation increases from 0·1 to 0·8.

As the mass increases from $3{\cdot}7 \,.\, 10^{34}$ gm. to infinity, the ratio of radiation-pressure to gravitation increases from 0·8 to 1·0.

The range $1{\cdot}8 \,.\, 10^{33}$ to $3{\cdot}7 \,.\, 10^{34}$ gm. corresponds to about 1 to 20 times the sun's mass, and we see that at the beginning of this interval radiation-pressure is only one-tenth of gravitation, and at the end it counterbalances four-fifths of gravitation. These figures correspond to complete ionisation (molecular weight 2); if we had taken only half the electrons to be broken off (molecular weight 4) the range would have been from $\frac{1}{4}$ to 5 times the sun's mass. The truth probably lies between these two assumptions.

I think it is probable that this result is something more than a coincidence, and that it points to the cause which determines why the stars are so nearly of one standard size. A gas-sphere under the influence of two opposed and nearly balanced forces would probably be on the verge of instability; we need only refer to the somewhat analogous case of rotating masses in which centrifugal force balances a considerable part of gravitation.* We should expect, therefore, that masses in which radiation-pressure counterbalances the greater part of gravitation would very readily divide under the influence of comparatively small disturbances—rotation, or perhaps imperfect homogeneity. Hence the primitive material would continually divide and subdivide until the masses were below the dangerous limit; after that there would be no further cause for division.

The suggestion is that though high radiation-pressure does not actually break up the mass, it produces a state in which the mass will, under ordinary circumstances, be broken up by rotation or other causes. Occasional large masses with exceptionally low angular momentum may survive, as in the case of Canopus. But when the mass is reduced to, say, half that of the sun, radiation-pressure lends practically no aid in breaking up, and smaller masses could only be produced in exceptional circumstances.

* *E.g.*, the Maclaurin spheroids become unstable when the centrifugal force is 0·36 of gravity at the equator; the Jacobi ellipsoids become unstable when the ratio varies from 0·19 to 0·55 at different parts of the equator.

17. *The Temperatures of Giant Stars.*—Although $(1-\beta)$ is now small, the equation

$$T_1^4 \propto g(1-\beta)$$

is not thereby rendered illusory.

Since β is independent of the density, the two principal results in which we previously found agreement with observation are obtained as before—the effective temperature varies as the sixth-root of the density, and the bolometric magnitude is independent of the stage of evolution so long as the star behaves as a perfect gas.

The total radiation varies as $M(1-\beta)$, and we can no longer consider this as practically proportional to the mass. By Table I., for a 9-fold increase of mass (from 0·5 to 4·5) $1-\beta$ increases 11-fold, so that in the range usually considered the total radiation varies more nearly as the square of the mass. This is the principal difference made by assuming ionisation. We see that the masses of stars must be even more uniform than was stated on p. 30.

This greater sensitivity to mass-changes enables us to account more readily for the occasional occurrence of exceptionally brilliant stars. Canopus is probably of magnitude $-5{\cdot}5$ or brighter, and certain evidence (such as Shapley's estimate of the parallax of the Hercules cluster) points to the existence of other stars of this brightness. With molecular weight 54 Canopus would have to be 180 times as massive as the sun; but with molecular weight 2 it need not be more than 40 times as massive—a much less violent assumption.

Taking the sun's absolute magnitude to be $5^{m}{\cdot}1$, the absolute magnitude of a giant star of the same type can be calculated. We obtain the following results *:—

TABLE II.

Mass.	Abs. Mag. m
0·5	+2·6
1·0	+0·7
1·5	−0·3
3·0	−1·7
4·5	−2·4

For any other type the absolute bolometric magnitude will be the same, but a small correction must be applied to obtain the absolute visual magnitude. In Table III. I give the necessary correction for objects which radiate like a black body; since the corrections are small, it may be hoped that the deviations of an actual star from a black body will not make any important difference.

The table is calculated from Nutting's † measures of the visual

* The middle result is practically adopted from observation, as already explained. The others follow from the values of $M(1-\beta)$.

† *Phil. Mag.*, 1915 Feb., p. 304.

intensity of light of different wave-lengths, together with Planck's radiation formula.

TABLE III.

Effective Temperature.	Ratio of Luminosity to Total Radiation.	Visual—Bolometric Magnitude.
2540°	0·092	+2·59
3000	0·206	+1·71
3600	0·417	+0·95
4500	0·723	+0·35
6000	1·000	0·00
7500	0·985	+0·02
9000	0·893	+0·12
10500	0·749	+0·31
12000	0·616	+0·53

The numbers in the last column are to be added to the absolute magnitude (for the mass in question) given in Table II.

18. *The Temperatures of Dwarf Stars.*—With a high molecular weight it was found that the whole dwarf series must be comprised within densities 0·7 to 1·5; but with molecular weight 2 the range is extended in both directions. Maximum temperature occurs at densities 0·2 to 0·4 according to mass, and it is no longer necessary or possible to assume that the sun is compressed to the maximum possible density. Thus we can have dwarf stars with densities up to 3 or 4. The temperature-density curve becomes more open, and it is possible to follow the relations some way down the dwarf series.

I have thought it worth while to work out in detail the temperatures near the maximum, and as far as practicable down the dwarf series. It is true that in this (as in some other places) I am pressing the theory further than it could be expected to go. But it may be interesting and suggestive to follow out fully the behaviour of a theoretical model; and since a comparison with observation is often possible, we may seek to find out whether the model is defective.

I have represented the deviations from a perfect gas by van der Waal's equation

$$p = R\rho T(1 - \rho/\rho_0)^{-1},$$

the term $a\rho^2$ usually added to p being negligible at the high pressures here considered. The equation is, of course, not accurately satisfied by any gas, but it seems likely to represent sufficiently well for our purpose the kind of deviation from Boyle's law which most concerns us. In order to obtain the correct temperature for the sun, it is found necessary to take ρ_0 to be 3·9. The constants have thus been adjusted to agree with observation at two points, viz. to give a temperature of about 6000° for the sun, and 6500° for a star of mass 1·5 and density 0·002.

The method of calculation was explained in § 10, but I have now found it much simpler to proceed from the centre of the star outwards; this avoids a series of adjustments by trial and error. Starting with an arbitrary central density and an arbitrary value of β, we can always build up a possible star; its mass and mean density are found at the end of the calculation. The density is unalterable; but we can afterwards change the mass to whatever value we please, changing β so as to keep $\frac{1-\beta}{\beta^4 M^2}$ constant (*cf.* equation (44)). Thus β can be found for stars of that particular density and of any assigned mass. The temperature is then given by (42). In this way I have obtained Table IV.

TABLE IV.

Effective Temperatures of Stars.

Mass / Density.	0·2.	0·5.	1·0.	1·5.	3·0.	4·5.
2·11	...	2590°	3880°	4920°	7350°	9300°
1·53	...	3580	5360	6790	10080	12640
·97	2720	4650	6960	8770	12810	15690
·65	3140	5350	7950	9960	14210	17000
·33	3520	5980	8820	10900	14930	17320
·13	3710	6250	9060	10980	14370	16230
·035	3540	5910	8350	9900	12460	13830
·002	2410	4000	5550	6500	8020	8820

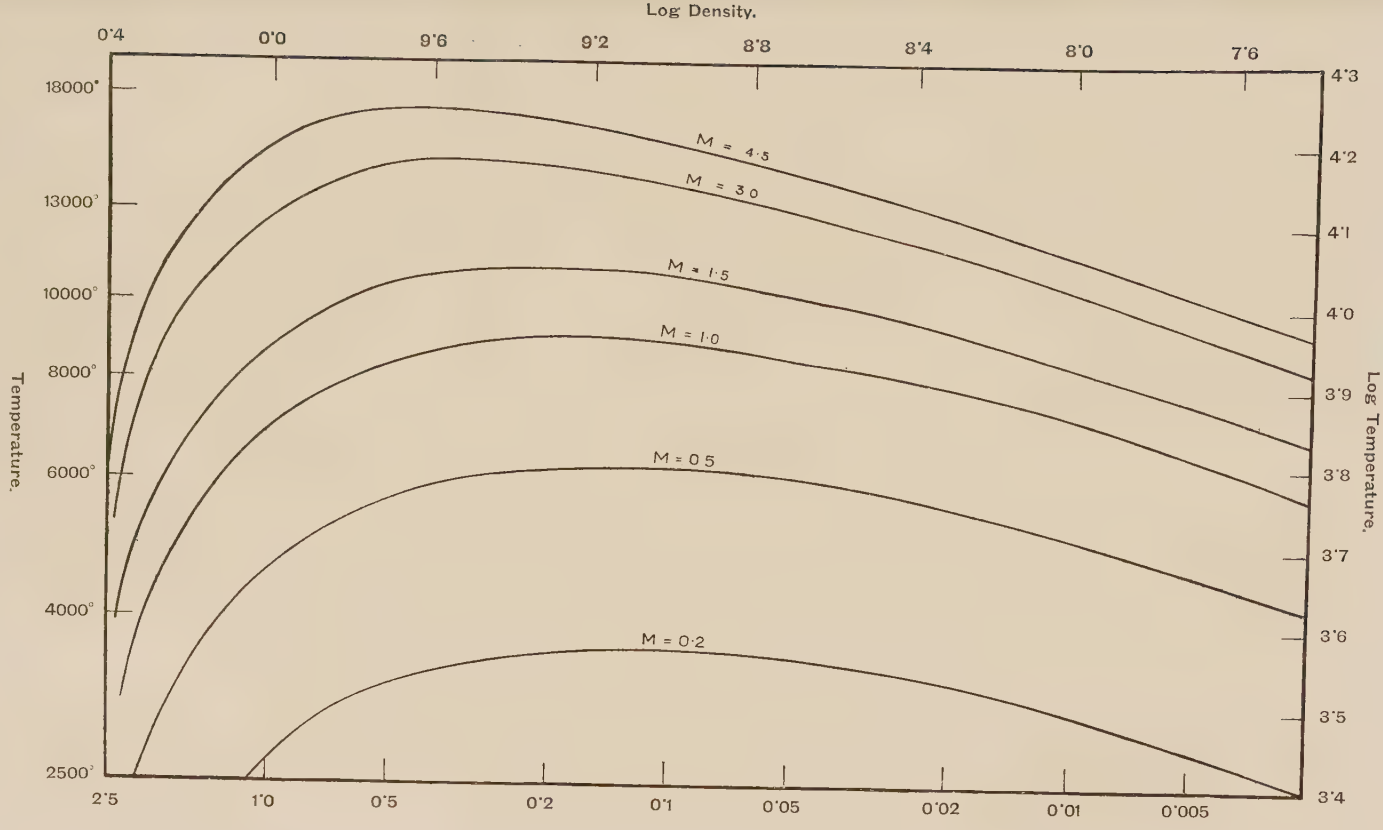
The masses are measured in terms of the sun as unit; the densities in terms of water.

The results are plotted in the diagram, Plate 10, in which the logarithms of the densities and temperatures are taken as abscissæ and ordinates.

The table illustrates H. N. Russell's contention that the small stars cannot rise to the high temperatures characteristic of the so-called earlier types. The maximum temperature attained varies nearly as the square-root of the mass. The following table (derived by interpolation) shows the mass necessary to attain a specified temperature:—

TABLE V.

Effective Temperature.	Minimum Mass.
14000°	2·5
12000	1·8
10000	1·2
8000	0·8
6000	0·46
4500	0·29
3000	0·14



Relation between Effective Temperature and Mass and Mean Density (illustrating Table IV.).

I had not anticipated so rapid an increase of maximum temperature with mass; but I believe that the observational evidence on the whole favours it.

It is easy to calculate from the curves the absolute luminosity of a star of mass M and effective temperature T. Take the density ρ from the curve; then the volume of the star is to the volume of the sun as $\frac{M}{\rho} : \frac{1}{1{\cdot}38}$; their surfaces are in the ratio $\left(\frac{1{\cdot}38M}{\rho}\right)^{\frac{2}{3}}$ and their total radiations in the ratio $\left(\frac{1{\cdot}38M}{\rho}\right)^{\frac{2}{3}}\left(\frac{T}{6000}\right)^4$. Converting this into magnitudes and applying the correction from Table III., we obtain the difference between the absolute visual magnitude of the star and that of the sun ($+5^{m}{\cdot}1$).

It may be noted that whereas the luminosities of the giants of given type vary approximately as M^2, the luminosities of the dwarfs cannot vary more widely than $M^{\frac{1}{3}}$, and even this variation is partly compensated by the corresponding changes of density. In the same spectral class the range of luminosity of the dwarfs ought therefore to be much less than that of the giants; but unless we use very fine divisions of spectral classification this uniformity of the dwarfs is likely to be masked by the great change in luminosity from type to type.

19. *Discussion of the Temperature-Density Curves.*—We proceed to deduce tentative conclusions from the curves. Where possible these are compared with observations; in other cases we must judge by the general reasonableness of the results.

(*a*) We calculate by this theory the difference of absolute magnitude of the dwarfs and giants of type M (temperature 3000°). I find the visual magnitudes to be—

Dwarfs $+10{\cdot}6$, $10{\cdot}6$, $10{\cdot}2$, $9{\cdot}9$, $9{\cdot}5$
for masses $0{\cdot}2$, $0{\cdot}5$, $1{\cdot}0$, $1{\cdot}5$, $3{\cdot}0$.
Giants $+4{\cdot}3$, $+2{\cdot}4$, $+1{\cdot}4$, $0{\cdot}0$, $-0{\cdot}7$
for masses $0{\cdot}5$, $1{\cdot}0$, $1{\cdot}5$, $3{\cdot}0$, $4{\cdot}5$.

Adams,* who has determined by the spectroscopic method absolute magnitudes of 10 dwarfs and 48 giants, gives an average magnitude $+10{\cdot}3$ for the dwarfs and $+1{\cdot}4$ for the giants. The greater range of the giants is also apparent in his results, viz.—

Dwarfs $+10{\cdot}7$ to $+9{\cdot}8$; giants $+3{\cdot}4$ to $-1{\cdot}0$.

41 out of his 48 giants were, however, between $+2^{m}{\cdot}0$ and $0^{m}{\cdot}0$, which on our theory would correspond to masses from $1{\cdot}2$ to $3{\cdot}0$.

Russell gives the average observed absolute magnitudes of the dwarfs and giants as $+9{\cdot}9$ and $+0{\cdot}6$ respectively.

(*b*) A star of mass less than $\frac{1}{7}$ of the sun could not, on this theory, attain an effective temperature 3000°, and hence all stars (or, at least, those falling into the usual spectral series) must have masses greater than $\frac{1}{7}$. The smallest masses of which we have

* *Nat. Acad. Sci. Wash.*, 1916, p. 157.

trustworthy knowledge are the fainter components of ζ Herculis and Procyon, from $\frac{1}{3}$ to $\frac{1}{4}$ the sun's mass; there is no evidence that any visible star has a mass so small as $\frac{1}{10}$ of the sun.*

Stars which reach a maximum temperature only a little greater than that of type M would tend to bridge the gap in luminosity between giants and dwarfs. The clear separation of the two groups seems to show that such stars must be rare (unless we can suppose that, owing to their rapid evolution, they have all reached a dense state). The giants and dwarfs become clearly separated at about type G, for which the minimum mass is 0·46; it is therefore probable that stars of mass less than half the sun's mass are comparatively uncommon. A theoretical reason for this was explained in § 16.

(*c*) The effective temperature for type B is rather uncertain, but perhaps we may take 13000° to correspond to B5. Hence by Table V. a star of type B – B5 must have mass 2·2 at least. It is well known that B stars are more massive than other types. Ludendorff † showed that the spectroscopic binaries of type B are on the average three times as massive as the other types. If the lower limit of mass for type B is 2·2 and for all stars 0·5, the proportion seems about right.

A star of mass 2·2 and type B5 will have absolute magnitude $+0^{m}\cdot9$, according to our curves. Since this is the least massive star of the type possible, it might perhaps be thought that $+0^{m}\cdot9$ would be the minimum absolute brightness of a B5 star; but this is not so. A B5 star of mass 3·0 will have a magnitude +1·4 or −0·7 according as it is in the later or earlier stage; for mass 4·5 the magnitudes are +1·4 or −1·6. There is a true minimum about $+1^{m}\cdot4$, and evidently there must be a large number of stars with magnitudes near to the minimum, since stars within quite a large range of mass have magnitudes differing from it very little. We may venture to predict that the frequency of luminosity for B5 will not be an error-curve, but will be more like half an error-curve, the frequency falling from the maximum to zero almost abruptly. Kapteyn ‡ has traced the frequency up to a maximum at absolute magnitude +0·9, but he found it impossible to determine what happened on the other side of the maximum.

(*d*) For a binary star a "hypothetical parallax" can be derived on the assumption that the mass of the brighter component is equal to the sun's mass. The luminosity deduced with this parallax may be called the "hypothetical luminosity." If H is the hypothetical luminosity, L the true luminosity, it is easy to show that $L = HM^{\frac{3}{2}}$.

For a giant star $L \propto M(1-\beta)$, so that $H \propto M^{\frac{3}{2}}(1-\beta)$. We

* Russell, *Nature*, vol. xciii. p. 256. Russell has just announced (*A.J.*, No. 711) for the companion of Krueger 60 a mass "between $\frac{1}{8}$ and $\frac{1}{9}$ of the sun's mass."

† *Ast. Nach.*, No. 4520. According to J. Halm (*Monthly Notices*, vol. lxxi. p. 636) the B stars are seven times as massive as the others, which would not fit our theory so well.

‡ *Ap. J.*, vol. xl. p. 82.

have seen that for high molecular weight the variations of $1-\beta$ are practically negligible, and in that case $H \propto M^{\frac{1}{3}}$. A range of mass from 0·5 to 4·0 would give a range of H from 1 to 2, or $0^m{\cdot}75$, in any one spectral class. The diagram given by Russell* shows that H is by no means so restricted. Some allowance must be made for accidental errors of various kinds, but the evidence seems to be irreconcilable with so small a range. With molecular weight 2, however, $(1-\beta)$ varies approximately as M, and H varies as $M^{\frac{1}{3}}$. The mass range 0·5 to 4·0 would then give a range of 3 magnitudes in H, which is about the amount indicated in Russell's diagram.

For the giants H increases with M, but for the dwarfs the relation is inverted and H diminishes as M increases. In the same spectral class $L \propto M^{\frac{3}{2}}\rho^{-\frac{1}{2}}$, and hence $H \propto \rho^{-\frac{2}{3}}$, and we see from the curves that for dwarf stars of given temperature the density increases with the mass. This enables us to make a rough calculation of the effective temperatures of the different types by assuming that the lowest H in each case (except for evident stragglers) corresponds to a certain maximum mass, say 4·5. If the mass is higher it will make very little difference except in type A, because the density is approaching its limiting value. From the diagram given by Russell I deduce in this way—

Type.	Faintest Hyp. Mag.	Effective Temperature.
A	3·2	10600°
F	4·7	7300
F5	5·0	6800
G	5·7	5800
G5	6·2	5300
K	6·9	4700
K5	7·5	4200

The table is only given in order to show that the progression of temperature is roughly of the amount expected.

(*e*) For double stars with accurate orbits and large parallaxes, whose masses and luminosities can be computed, it is possible to use the curves to determine the temperatures. In this way I find—

Star.	Adopted. Mass.	Adopted Abs. Magnitude. m	Calculated Temperature.	Observed Type.
Sirius	2·45	1·30	12800°	A0
Procyon	1·0	3·03	8300	F5
α Centauri	1·0	4·73	6400	G
	0·85	6·10	5100	K5

The parallaxes were taken from Kapteyn and Weersma's list, and the ratios of the masses of the two components from Boss's Catalogue.

* *Nature*, vol. xciii. p. 257.

The agreement of the last two columns is scarcely so good as might have been hoped (all the temperatures seem to be too high for the types).

A small uncertainty of the mass makes a rather large difference in the resulting temperature; hence for double stars of smaller parallax it seems better to reverse the procedure, and, adopting a temperature which is believed to correspond to the observed type, calculate the mass, and hence from the orbital elements deduce the parallax. The following results were obtained:—

Star.	Type.	Adopted Temperature.	Calculated M.	Calculated π.	Observed π.
		°		"	
γ Virginis	F	8000	1·05	·093	·058
δ Equulei	F5	7000	2·47	·046	·067
42 Comæ	F5	7000	1·04	·058	·062
η Coronæ	G	5800	0·54	·072	·076
ζ Herculis	G	5800	0·43	·151	·142
ξ Ursæ Majoris	G	5800	0·62	·152	·179
70 Ophiuchi	K	4700	0·36	·264	·212
85 Pegasi	G	5800	1·50	·067	·088

The orbital data and magnitudes were taken from a paper by E. Öpik (*Ap. J.*, vol. xliv. pp. 298–9). Four of the observed parallaxes are by Slocum (*Ap. J.*, vol. xli. p. 237), and the others are from Kapteyn and Weersma's list. The calculated masses refer to the brighter component.

The agreement of the last two columns is satisfactory; but it must be remembered that the ordinary "hypothetical parallax" is usually very close to the truth. The masses of all these stars are too near unity to give a satisfactory test of the theory.

20. *Energy of a Star.*—It is well known that the hypothesis that the sun's energy is derived from contraction leads to a value of the sun's age which is much too small to be reconciled with estimates of the earth's age based on studies of radioactive minerals and other geological evidence. Perhaps it has not been realised that the contraction hypothesis involves a rapidity of evolution of the giant stars which is from some points of view even more startling. I give a numerical instance worked out on the present theory, but on any other theory the order of magnitude of the results could scarcely be altered.

Consider the time taken for a giant star of mass 1·5 to rise from effective temperature 3000° to 6000°, *i.e.* from type M to G. At 6500° the energy of contraction Ω was found (p. 23) to be $1{\cdot}18 \times 10^{48}$ ergs, and since $\Omega \propto T^2$ we find $\Omega_{6000} - \Omega_{3000} = 7{\cdot}54 \times 10^{47}$ ergs. Of this, the fraction $\frac{1+\beta}{4}$ is required for ætherial and translational molecular energy in the star, and therefore, taking the new value of β (0·826), we have at most $4{\cdot}1 \times 10^{47}$ ergs available for radiation. The actual amount is less than this,

since some energy must be used in increasing the ionisation and internal atomic energy.

Taking the solar constant to be 1·93 gram-calories per sq. cm. per minute, we find that the sun radiates $1{\cdot}20 \cdot 10^{41}$ ergs per year. The giant star in question is 5·4 magnitudes brighter (bolometrically), and therefore radiates 145 times as fast. Hence the time taken to rise from 3000° to 6000° is not greater than $4{\cdot}1 \cdot 10^{47} \div (145 \times 1{\cdot}20 \cdot 10^{41})$, or 24,000 years!

We can only avoid a short time-scale by supposing that the star has some unknown supply of energy. Probably the simplest hypothesis of this kind is that there may be a slow process of annihilation of matter (through positive and negative electrons occasionally annulling one another). This would set free an almost inexhaustible store of energy. But the defect of all such processes is that they supply energy at some fixed rate—unlike the energy of contraction, which is drawn on as required and automatically replaces the amount radiated. A star would soon expand or contract until the rate of radiation balanced the fixed rate of supply. But in that case it is difficult to understand why the rate should be so very different in different stars of similar mass.

The usual view that evolution takes place in the direction of continually increasing density has practically no foundation, unless we believe that the energy comes mainly from contraction.

On the contraction hypothesis the question whether in the giant stage evolution proceeds more rapidly in heavy stars or light stars depends (according to the present theory) on the molecular weight. If r is the radius of the star—

$$\text{Energy of contraction} \propto \frac{M^2}{r}.$$

$$\text{Rate of radiation} \propto M(1-\beta).$$

$$\text{Effective temperature} \propto M^{\frac{1}{2}} r^{-\frac{1}{2}}.$$

Hence, assuming that a constant fraction of the energy of contraction is available for radiation,*

$$\text{Time for a given rise of effective temperature} \propto M^{\frac{1}{2}}/(1-\beta).$$

This increases with M when the molecular weight is high, but decreases with increasing M (up to a certain limit) for molecular weight 2.

There is thus a possibility that during the giant stage heavy stars may change type more rapidly than light ones; but, nevertheless, the light stars have, as it were, not so far to go, and may still reach the dwarf stage first.

Summary of Conclusions.

1. If the mass-coefficient of absorption is the same for all stars, the effective temperature varies as the sixth-root of the

* The fraction available for radiation is $(3-\beta)/4$, *less* the unknown internal atomic energy. Our ignorance of the latter renders the conclusion somewhat uncertain.

density, and the total radiation is independent of the stage of evolution, so long as the star is in a perfect gaseous condition. Conversely, the approximate confirmation of these laws by observation seems to show that the absorption is nearly the same for all stars.

2. Owing to the compensatory effect of radiation-pressure the distribution of temperature in the interior depends only to a very small degree on the molecular weight of the material or the amount of ionisation. For a giant star of type G the central temperature is of the order 5,000,000°.

3. Assuming the highest possible degree of ionisation in the interior, the mass-coefficient of absorption is 5·4 C.G.S. units. If the coefficient depends on the temperature, this value refers to approximately the highest temperature in the interior. The absorption cannot anywhere exceed 30 C.G.S. units, except possibly for a very limited range of temperature.

4. The actual masses of stars correspond to the range in which radiation-pressure increases from trifling to predominant influence. Assuming that a gas-sphere in which gravitation is largely counterbalanced by radiation-pressure would be in a state approaching instability, and would readily break up under small rotational velocity or other causes, it is evident that the matter of the universe must aggregate into bodies of mass agreeing quantitatively with the observed masses of stars.

5. With low ionisation the total radiation of a giant star varies approximately as the mass, with high ionisation as the square of the mass. In either case the observed range of luminosity of the giants indicates a very close uniformity of mass of the majority of the stars.

6. Determinations of the temperatures of stars too dense to behave as a perfect gas involve more doubtful assumptions. Assuming a high degree of ionisation, a theoretical model has been found which appears to represent the chief results of observation satisfactorily. According to this, a body must have at least one-seventh the sun's mass in order to attain an effective temperature of 3000°; stars with mass less than 2·2 times the sun's mass will not reach the B stage. The maximum temperature is attained when the mean density is between 0·2 and 0·5. The distribution of the luminosities of the B stars may be expected to be strongly asymmetrical, the maximum frequency corresponding to minimum luminosity.

7. If contraction is the only source of energy, the giant stage of a star's existence can scarcely exceed 100,000 years.

Errata in the First Paper.

P. 19, formula (8*a*), *read* "amount flowing *inwards* is $\pi A - \frac{2}{3}\pi B$."

P. 21, line 3 from bottom, *for* $\frac{4}{3}$ *read* $\frac{16}{9}$.

Note on the Southern Magnitude Distribution, with special reference to the Perth Astrographic Zone. By Professor R. A. Sampson, D.Sc., F.R.S. (Plates 11, 12.)

In a recent publication of the National Academy of Sciences,* Mr. Seares concludes that the galactic condensation of small stars arrived at by Chapman and Melotte from a study of the Franklin-Adams charts † must be considered altogether erroneous, and that the truth is to be found much nearer to the older estimates which the counts of Chapman and Melotte displaced. The divergence is extreme; at mag. 16·3 the respective measures are:—Kapteyn, 32; Chapman and Melotte, 4·2; Seares, 21·4. With these Seares compares also Herschel's estimates for mag. 14, viz. Herschel, 12·9, where Kapteyn gives 11 and Chapman and Melotte 3·9. Seares' publication is, so far, preliminary only, but it is based upon counts of some 40,000 stars, in eight galactic zones, down to mag. 17·5 on the Mount Wilson photographic scale, equated to 16·3 of the Groningen visual scale. The totals only are available, not the counts of the separate magnitudes. Chapman and Melotte's counts, if I gather correctly the method described, depend upon some 50,000 or 60,000 stars in specimen areas, and are grouped according to magnitudes. Until a contradiction of this nature is cleared up, we know nothing at all of the galactic condensation. Its importance is my excuse for offering a few remarks which do not contribute directly to its solution, but touch it collaterally through the allied question of using the Chapman and Melotte scale as representative of any chance zone upon the sky, say one of the astrographic zones. I suppose that only by comparing together all possible collateral evidence shall we at length reach a reliable magnitude scale. My attention was turned to the matter by the valuable work lately done by Professor Turner in insisting that the magnitude scales of the various astrographic zones should be collated, and showing that, even from the material at hand, remarkable and consistent evidence was forthcoming of systematic "obscuration" of a large area of the sky, in the relative deficiency of faint stars compared to bright ones.‡ Among the zones which he employs is that of $-32°$, taken and measured at Perth, W.A.§ Mr. Cooke's magnitude scale is professedly provisional only,‖ but the following remarks seem to me to show that it deserves careful attention. His method was to expose upon a 7-mag. or 8-mag. star for various periods, viz. 1^s, $1^s{\cdot}4$, 2^s, . . . 64^s, each period rising with the ratio $\sqrt{2}$ to the last, first at the centre of the plate, then at the middle of one side, and then at one of the corners. These images were then brought side by side, named A, B, . . . N, and

* F. H. Seares, *Proc. Nat. Acad. Sci. U.S.A.*, vol. iii. (1917), p. 217.
† *R.A.S. Mem.*, vol. lx. (1914), p. 145.
‡ *M.N.*, vol. lxxv. (1915), p. 480.
§ *Ibid.*, p. 144.
‖ *Perth Astrographic Catalogue* (1911), vol. i. Introduction.

turned into an eye-piece scale, with which the appearance of any star in any part of the astrographic plate might be compared with assurance of uniformity. The estimates of magnitude are published accordingly under the letters A, . . . M,—N being invisible; only a provisional attempt is made to translate these into numbers, but by identification with stars in the C.P.D., Mr. Cooke gives the value of 1 step as 0·3 mag., and the magnitude of the star D as 9·3.

We may note that the total number of stars in this zone is upwards of 60,000; therefore, as a piece of classification, it is upon the same scale as the counts of Seares, and Chapman and Melotte, and more satisfactory in some respects, because complete and continuous within its range; but being confined to a single zone of declination, it is not intentionally representative of the whole sky. But suppose for the moment that it is representative of the whole sky, then the values of the symbols A, B, . . . may be derived from comparison with Chapman and Melotte's numbers of the average numbers of stars of each magnitude per square degree. Professor Turner has made this calculation, and shows that it would follow that the Perth star D = 10·08 mag., while 1 step = 0·52 mag.* I wish to point to some considerations that make it almost certain that Mr. Cooke's provisional estimates are much nearer to the truth. First, with regard to the value of the step. Stars of unknown intensity, A, B, . . ., exposed for uniform time T, are equated to images of a standard star Ω, exposed for various times t_A, t_B, ... If we take Schwarzschild's actinic law as applicable, we have

$$A^q T = \Omega^q t_A,$$

in which experiment gives q pretty consistently the value 1·25. Hence if m_A, m_B, . . . are the associated magnitudes,

$$m_A - m_B = -\frac{2{\cdot}5}{q} \log_{10} (t_A/t_B),$$

so that, if $t_A/t_B = 1/\sqrt{2}$,

$$m_A - m_B = -\frac{2{\cdot}5}{q} \times 0{\cdot}15,$$

$$= -{\cdot}30, \text{ for } q = 1{\cdot}25.$$

In order to make $m_A - m_B$ different from this, say equal to $-0{\cdot}5$, we should require $q = 0{\cdot}75$, which is quite at variance with experience.

Next, with regard to the zero point of the scale. Mr. Cooke's comparison with the C.P.D., though derived from "a few plates, taken at random," is very fairly consistent, giving the following equations:—

A = 8·4 mag.	E = 9·7 mag.
B = 8·7 ,,	F = 10·0 ,,
C = 8·8 ,,	G = 10·2 ,,
D = 9·3 ,,	H = 10·3 ,,

* *Loc. cit.*, p. 144.

One may conclude that to take D = 9·3 mag., with interval 0·3 mag., makes Cooke's scale identical with that of the C.P.D. Now the C.P.D. was adjusted generally to Gould's Zone Catalogue. Hence, if Cooke's D should really be considered as 10·1 mag., we should conclude that the limit of the Cordoba observers, which they give as 9½ mag., with a few of 10 mag., should really be 10½ mag. or more. But as they used only a 5-inch transit circle, I find it very hard to suppose that they habitually observed 10½-mag. stars, with occasional 11 mag.

Nor again can one suppose that the second Perth exposure, which was 2 min. only, and was intended to secure everything of 11 mag., but not more, with a second image, really reached beyond 13 mag., as the comparison given by Professor Turner (*M.N.*, vol. lxxv. p. 144) would require.

I conclude that the Chapman and Melotte numbers do not in the least represent the actual distribution in the Perth zone − 32°. But I would interpolate here that this conclusion does not detract in any way from the use subsequently made by Professor Turner of the Perth counts.

But it is permissible to go a little further and ask how many steps must the Perth counts be extrapolated before they give the totals per square degree which Chapman and Melotte find as an average for the whole sky down to 17 mag. It is easily seen that this is reached some 7 or 8 steps beyond Cooke's K, or, according to Cooke's scale, between 13 mag. and 14 mag. No doubt there is little certainty about such an extrapolation, but it seems to point to an irreconcilable divergence between the two sequences. But the zone − 32° is not especially rich in stars. We may consider the following numbers, representing the logarithms of the average number of stars per square degree recorded for various zones; they are quoted from Professor Turner's papers referred to, adding only Perth − 38°, counted at Edinburgh, and Greenwich.

Zone.		log N.
Greenwich,	+87° − 63°	1·541
Vatican,	+64°, 62°, 60°	1·966
Oxford,	+28°	2·040
Bordeaux,	+17°	1·853
Algiers,	+ 1°	1·689
Perth,	−32°	1·981
,,	−38°	2·013
Cape,	−41°	2·094

Therefore it seems to me that, generally, the material of the *Astrographic Catalogue* makes it difficult to accept the Chapman and Melotte averages as representing the actual mean distribution over the whole sky.

In making the foregoing remarks upon the substantial justice of Cooke's scale, as derived from the C.P.D., I would not overlook

the paper recently published by Dr. Halm,* of which one section deals with the corrections to the C.P.D. scale. These he finds to be in the same sense and of even greater amount than would bring Cooke's magnitudes into agreement with Chapman and Melotte's, running, namely, as follows:—

C.P.D.	Standard.	Standard *minus* C.P.D.
m	m	m
7·6	7·40	−0·15
8·0	8·50	+ ·49
8·4	9·29	·86
8·7	9·78	1·04
9·0	10·22	1·22
9·3	10·66	1·37
9·5	11·06	1·53
9·7	11·46	1·73
9·9	11·93	+2·00

It is unnecessary to dwell upon the complete rearrangement of scale that such corrections would imply; they are brought out sufficiently above, in the consequences of comparing the Perth zone with Chapman and Melotte. And I would hesitate to criticise a work which is evidently the outcome of a great deal of thought and labour, and is at present only published in a more condensed form than belongs to such fundamental work. But I must draw attention to one feature of it, to which Dr. Halm does not expressly devote any remarks, though in other systems of fundamental photographic magnitudes it has occupied the cardinal place—I mean, care that the quantities measured represent the whole actinic effect of each star, and do not, for example, progressively fall off as the images grow fainter. This would seem to be particularly required where the scale reaches out beyond the point where any other standard investigations have been made.

In the astrographic work it is usual to "collect" the diffused edges of an image and add them to the centre in forming an estimate of diameter. Whether this leads to uniformity may be doubted, especially where the images are not round. Dr. Halm follows another plan: he measures the diameter of the completely blackened nucleus. This, deliberately, is not the whole actinic effect. Its advantages are a matter for experience. The standards are obtained by exposures upon a chosen star (σ *Octantis*, taken at photographic magnitude $m_0 = 5{\cdot}73$) exposed for a series of periods from about 1^s to 1500^s, giving a curve relating diameter and time (t_n); then the equation is employed (see p. 614 above),

$$m - m_0 = \frac{2{\cdot}5}{q} \log (t_m/t_n),$$

* "A System of Photographic Magnitudes for Southern Stars," *M.N.*, vol. lxxiv. (1914), p. 600.

where t_m is the exposure of the unknown star m, and the unknown magnitude m is derived from it, by equating its measured diameter with one of these shown by the standard star m_0. Thus, for example, with 1000^s exposure of the plate (m) we should have the following data *:—

d.	$\log t_n$.	$\log (t_m/t_n)$.	$m - m_0$.	m.	$\sqrt{d}$.
0·92	2·87	0·13	0·27	6	0·96
·80	2·38	0·62	1·27	7	·89
·67	1·89	1·11	2·27	8	·82
·57	1·40	1·60	3·27	9	·76
·48	·92	2·08	4·27	10	·69
·39	·42	2·58	5·27	11	·62
·32	− ·06	3·06	6·27	12	·56

I have added the last column to show that the result is, apparently exactly, a linear relation between m and $\sqrt{d}$. My criticism of this method enters where the ratio of the exposure times t_m/t_n is large. We have seen above that $m = 12$ (p. 616) is required in order to reach 10 mag. on the C.P.D. scale. For this end the standard star σ *Octantis* is exposed for about 1^s, and the star investigated for about 1000^s. One would suspect that in this time, owing to atmospheric unsteadiness, the image of the fainter star would wander a sensible amount, and its completely blackened nucleus appear systematically smaller in proportion to its whole actinic effect than that of the standard star which was only exposed for a thousandth part of the time.

As against this, a control of the results is given by comparison with Schwarzschild's *Aktinometrie* † down to mag. $m = 9{\cdot}4$, or with a ratio of exposure times 1 : 30. In the same way Cooke's Perth scale, considered above, which is similar in principle, shows no stretching of magnitudes over a similar ratio of exposure times. It is in the extension from 30 times to 1000 times required to pass over the next three magnitudes that I would suggest a doubt. For example, if we accept Dr. Halm's corrections to the C.P.D., it would follow that the Cordoba observers with a 5-inch instrument habitually took 11-mag. stars, with an occasional 12-mag., and I find this difficult to believe.

I come now to another point. The plates for zones $-38°$, $-39°$, $-40°$, taken at Perth, were measured at Edinburgh. I have recently had counts made of these, with special reference to their bearing on Professor Turner's "obscured patch," or region in which small stars are relatively deficient. The demonstration of this region is essentially cumulative, and gains very much strength from the variety of material and treatment through which it

* Derived from Table II., Halm, *loc. cit.*, p. 604.
† Halm, *loc. cit.*, p. 606.

appears. Our magnitudes were not taken with Cooke's scale, but by estimating the collected diameters in the method indicated above, in all by six different groups of measures, viz.: 1, S.A.F. and I.C.F; 2, E.W. and L.C.; 3, R.W.W.; 4, E.K. and J.M.; 5, G.C.; 6, J.S.

The counts were made separately over the centre of the plates, that is, between réseau divisions 5 and 23, and the margins beyond this, having areas respectively $\left(\frac{18}{26}\right)^2 = \frac{81}{169}$ and $\frac{88}{169}$ of the whole, in order to examine any systematic effect there might be due to inferior definition at the edges.

When the results were examined, it appeared that only for zone $-38°$ were they so regular in estimates of diameters as to allow one to treat them with confidence hour by hour as parts of one whole. This zone was measured throughout, with trifling exceptions, by 1, S.A.F. and I.C.F. The other zones were much broken up among the six measurers, and the estimates of diameter are occasionally strikingly discordant. In considering the results, I am so much impressed with the superiority of Cooke's method, and with the importance of good magnitude estimates of the plates, as well as good measures of position, that I have decided to re-estimate the whole three zones with his focal scale, provided some test studies now being made seem to justify it. Meanwhile, what is given below depends chiefly upon zone $-38°$. There remained, however, another test besides the diameters, namely, the relative frequency of stars with a third image. This would seem to be much less exposed to personality than the estimates of diameter, depending as it does only on detection of the third image. The results were easily collected, and I give, in fig. 1, Plate 11, the charts of the logarithms of the ratio of the whole number of stars to stars showing a third image, for each of the three zones for each hour of R.A. The numbers 1 to 6, attached, indicate the measurers.

This chart speaks for itself; it may be said, generally, that the most promounced features or breaks are those which are confirmed by all three, and that they do not seem to be associated with change of measurer, or present any obvious relation to an annual term. Take, for instance, the remarkable oscillations between 13^h and 24^h. The best measured of the zones ($-38°$) is also the smoothest. Professor Turner's patch is shown by it quite clearly, running from 23^h to 6^h. To recognise its occurrence here in confirmation of proofs given elsewhere is, of course, a very different thing from claiming its reality upon the evidence of this chart alone.

The following chart (fig. 2, Plate 12) relates to zone $-38°$ alone. The logarithms of the ratios are given separately for whole plate and central portion—(1) for third-image stars compared to the whole; (2) diameter 24 compared to diameter 12; and (3), diameter 16 compared to the whole. The magnitudes associated with these are approximately as below, upon the Perth scale, and upon what

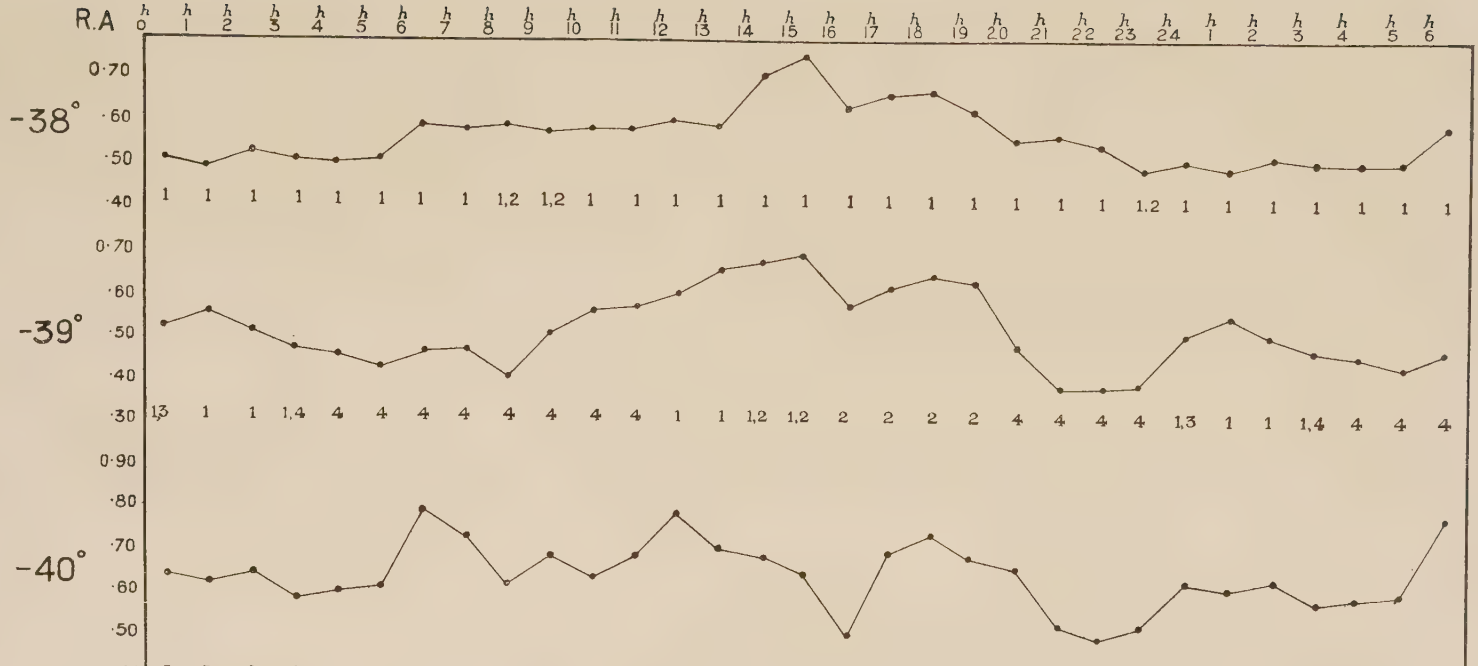


Fig. 1.—Log Ratios of Numbers of Stars with third Image to whole

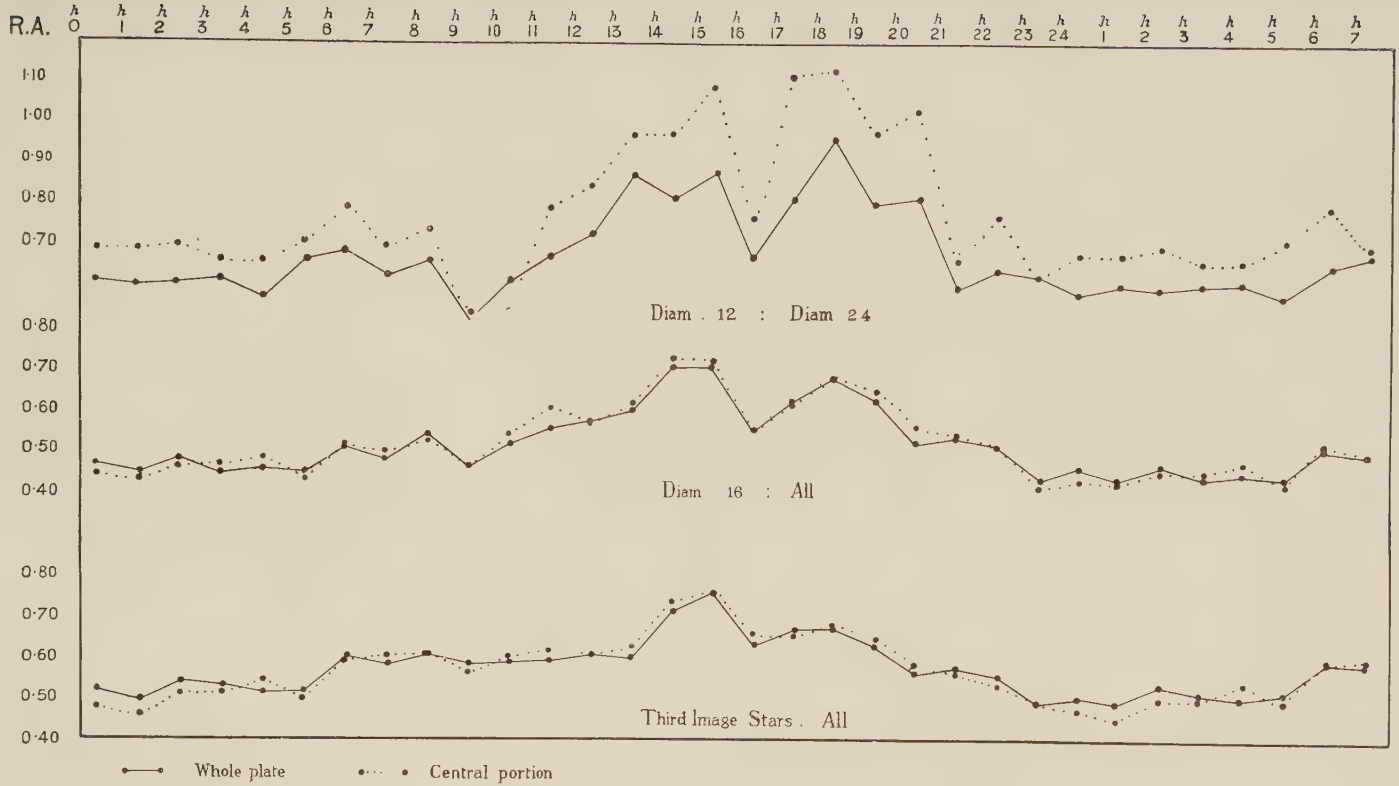


Fig. 2.—Log Ratios of Numbers of Small and Large Stars.

for convenience I have referred to as the Chapman and Melotte scale respectively :—

	Perth.	C. and M.
	m	m
Third image	10·0	11·4
Diam. 24	9·6	10·7
,, 16	10·1	11·5
,, 12	10·7	12·0
Whole = Diam. 6	11·5	13·1

The following are the numbers represented upon the chart, with notes as to the region of the sky with respect to the galaxy, and the dates when the plates in question were taken :—

	Whole Plate.			Central Portion.				
R.A.	log (all /3rd image).	log (12 diam. /24 diam.).	log (all /16 diam.).	log (all /3rd image).	log (12 diam. /24 diam.).	log (all /16 diam.).	Region.	Dates of Exposures.
0h	·52	·61	·47	·48	·68	·44	A	1911 Dec. 9, 12, 18.
1	·50	·60	·44	·46	·68	·43	,,	1911 Dec. 9, 12.
2	·54	·61	·48	·51	·70	·46	,,	1911 Dec. 12, 18.
3	·52	·62	·45	·51	·66	·46	,,	1911 Dec. 18, 1912 Jan. 14.
4	·51	·58	·46	·54	·66	·48	,,	1912 Jan. 14, 21.
5	·52	·66	·44	·50	·72	·43	,,	1912 Jan. 21, Feb. 22.
6	·60	·68	·51	·59	·79	·52	,,	1912 Feb. 22, Mar. 20, Apr. 10.
7	·59	·63	·49	·60	·70	·50	B	1912 Apr. 10, 14.
8	·60	·67	·54	·61	·74	·53	,,	1912 Apr. 10, 12, May 10.
9	·58	·53	·47	·56	·54	·46	,,	1912 Apr. 10, 14.
10	·59	·62	·53	·60	·55	·55	,,	1912 Apr. 10, May 8, 10, 1913 June 8.
11	·59	·68	·56	·62	·79	·61	C	1912 May 5, 8, 10.
12	·61	·73	·58	·61	·85	·58	,,	1912 May 5, 10.
13	·60	·88	·61	·63	·97	·63	,,	1912 May 5, July 21, Aug. 20.
14	·72	·82	·72	·74	·97	·74	,,	1912 July 26, 1913 July 22.
15	·76	·89	·72	·77	1·09	·73	D	1912 July 26, 1913 Aug. 7.
16	·64	·67	·56	·67	·77	·57	E	1911 Aug. 20, Sept. 14, 1912 July 18, 26.
17	·67	·81	·63	·66	1·12	·62	F	1911 Aug. 20, 1912 July 24, Aug. 12.
18	·68	·97	·69	·68	1·13	·68	,,	1912 Aug. 12, 1913 Aug. 20, Sept. 1.
19	·64	·80	·63	·65	·97	·66	G	1912 Oct. 12, 1913 Sept. 1.
20	·57	·82	·53	·59	1·03	·56	,,	1912 Oct. 12, Sept. 19.
21	·58	·60	·54	·57	·67	·55	A	1911 Oct. 25.
22	·56	·65	·52	·54	·77	·52	,,	1911 Oct. 25, Nov. 15.
23	0·50	·59	·43	·50	·63	·42	,,	1911 Nov. 12, 15, Dec. 9.

A, non-galactic, poor in stars ; B, a faint part of galaxy ; C, non-galactic, but fairly rich ; D, bright galactic region ; E, a dark region in galaxy ; F, bright galactic region ; G, fairly bright, bordering galaxy.

The first comment upon these charts is to the effect that restriction of counts to the central region, in which the star images are perfect and fainter stars are not lost by diffusion of their light, makes, generally speaking, no difference. The next most striking feature is the galactic condensation; it is perhaps emphasised by the remarkable way the dark rift in the middle of it, about 16^h, shows itself by an indentation of the curve. For the rest, the only definite feature is Professor Turner's obscured patch, extending from 23^h to 6^h in this zone. It is unmistakably present in just the position which he derived from analysis of the zones on each side, Perth $-32°$ and Cape $-41°$.

In conclusion, I give the actual totals of stars N_m, counted down to and including each diameter upon zone $-38°$.

Diameter.	N_m.	m.	Residual.
50	536	8·10	−0·07
40	1462	8·94	− ·13
30	2971	9·91	+ ·14
28	3359	10·12	+ ·23
26	3950	10·34	+ ·28
24	5298	10·57	+ ·21
22	6802	10·80	+ ·19
20	9409	11·05	+ ·11
18	13389	11·32	− ·03
16	18586	11·58	− ·14
14	22305	11·88	− ·06
12	26753	12·20	+ ·05
10	39331	12·53	− ·09
8	51683	12·92	− ·01
6	68527	13·35	+ ·01

To test the smoothness of these the corresponding magnitudes are calculated from Professor Turner's tables of Chapman and Melotte's scale, and reduced to the formula

$$m = 16^m{\cdot}12 - 1^m{\cdot}14\sqrt{d}.$$

The corresponding magnitudes found are shown in the column m, and the residual excesses over the Chapman and Melotte numbers corresponding to the counted stars in the adjoining columns. A distinct undulation is shown in the curve, due perhaps to a form of "decimal equation" or similar personality in the measurers 1, S.A.F. and I.C.F., and comparing somewhat unfavourably, *e.g.* with the numbers shown by Professor Turner,* in putting Cooke's

* *Loc. cit.*, p. 145.

Perth estimates for zone -32° from comparison images alongside the same standard, when the mean discrepancy of the first nine values is $\pm 0{\cdot}04$ mag.

I am indebted to Mr. E. A. Baker and Miss I. C. Falconer for the actual counts and statistical collections of zones -38°, -39°, -40°, together with a great deal of associated work that does not appear in this paper.

Sunspots in a High Southern Latitude. By Mrs. Walter Maunder.

(*Communicated by the Astronomer Royal.*)

Since 1910 the Greenwich heliographic register has been very largely supplemented from the Cape Observatory. For the earlier years, as the slight activity of the sun made it possible to measure the solar photographs closely to date, the Greenwich photographs were used whenever available, and the gaps were filled from those sent later from the Cape. Since the war, however, the reduction of the staff, combined with the increase in the solar activity, caused the measuring to fall somewhat into arrear, and the Cape photographs arrived before the Greenwich photographs were selected, and Mr. Newton, who was in charge of the department, had frequently four photographs to choose from for one day instead of two.

One consequence of this was that, in examining the Cape negatives, Mr. Newton detected a large number of very small faint markings which for distinctness I will call "flecks,"—markings that are certainly more than "pores," if not always quite of the order of "spots," though no rigid division can be drawn between them and the more obvious formations. None but an observer as keen and careful as Mr. Newton would have detected them in the first place, but also there seem to be two reasons for the fact that these "flecks" were to be discovered upon the Cape negatives rather than upon those taken at Greenwich. First, when possible, a considerable interval of time is allowed to elapse between the taking of the two daily solar photographs at Greenwich—it may be twenty minutes or an hour or more; at the Cape the photographs are taken immediately following each other, with usually but a minute's interval. No "fleck" was admitted as a "real" marking unless it was found on two photographs, but as it is evident that they are very short-lived as a class, it is clear that though they may survive for a minute or two, they may have vanished in a quarter of an hour. The second reason is more obscure, for there is no doubt that this order of marking does appear more clearly on the Cape photographs than on the Greenwich, even where the definition on the latter is as good or better than on the former. It may be a question of exposure and development as between the two observatories; it may be a question of

photographic level and focus which makes these faint markings show a stronger contrast to their surroundings on the Cape negatives.

So long as these "flecks" were found only within the usual sunspot latitudes Mr. Newton paid them no particular attention; they were simply examples of very faint and short-lived spots. But in the spring of 1916, when examining the photographs of 1915 December, he found a similar faint marking on the Cape negative of 1915 December 26 which was evidently quite beyond the received limits of sunspot latitudes; it was evidently a genuine solar marking, as it was clearly visible on two solar photographs, but as these were taken at an interval of about a minute, it may have lasted for but a short time; it was not to be detected either on December 25 or 27. When measured and reduced it gave for its heliographic longitude and latitude 63°·9, −59°·5, a latitude which constituted a record beyond all doubt: the highest registered hitherto was 50°·4 by Professor Peters in 1846. The number of Mr. Newton's spot in the Greenwich register is 7578, and another similar high latitude spot is No. 7486 of 1915 October 21, in hel. long. 143°·6 +59°·7.

But both of these were evanescent, and until such a "fleck" was detected which persisted for a day or longer, it remained doubtful whether these high latitude markings belonged really to the order of spots or merely to that of the "pores" that characterise the sun's surface everywhere. It is the unfairness of the fortune of war that it should fall to my lot and not to Mr. Newton's, to whom I might say they rightly belong, to bring before the Society an instance which gives us the right to assign to them the rank of sunspots.

A few days ago, when examining the reductions of the photographs for 1916 March, I found that one "fleck" was in south latitude 63°·0, longitude 125°·6, on March 10, and on March 14 I found again a "fleck" in south latitude 63°·3 and longitude 119°·6. These were so close in position that it is evident that if they were not the same actual spot they belonged to the same disturbed area, and I looked up at once not only these two photographs, but also the intermediate days and the duplicate photographs for the whole period. The sun's south pole was then most fully presented to us, which probably accounts for their more easy detection. I have remeasured this region of the sun on all the photographs available for the period 1916 March 8 to March 15 inclusive, rejecting all markings that did not seem to be registered on at least two photographs. The following table gives the positions in three different latitudes of areas that are distinctly disturbed, *i.e.* in latitudes −63°, −65°, and −72°. It will be noted that there is considerable uncertainty in the positions measured on photographs which are quite close together in time, and this may be due partly to the difficulty of measuring such faint indefinite markings on photographs which differ greatly in definition and contrast; partly to the fact that these "flecks" are so close to the

limb that they are seen strongly foreshortened and may easily be covered in part by faculæ at different times. In these regions, too, a very small difference in the position of a spot as measured, necessarily means a considerable difference in the latitude and longitude as computed. But it is evident that markings, not easily to be distinguished from spots, can persist in very high latitudes; but the evidence is not yet sufficient to find whether the individual spots lie in polar "rings" or "streams" or whether they have a steady drift, or again whether they are subject to abnormal proper motions.

Sun Flecks in High Southern Latitudes, 1916 March.

Date 1916.	No. and Place of Photograph.	G. Civil T.	Heliographic Co-ordinates of Fleck.		No. and Place of Photograph.	G. Civil T.	Heliographic Co-ordinates of Fleck.	
			Long.	Lat.			Long.	Lat.
		h m s	°	°		h m s	°	°
March 8	18616 G.	11 59 20	143·2	−72·2	18617 G.	12 2 14	143·7	−72·2
9	3832 C.	7 50 42	136·4	−72·2	3833 C.	7 51 32	136·7	−72·4
10	3834 C.	8 10 28	120·2	−73·3	3835 C.	8 11 28	120·8	−73·3
11	3836 C.	8 19 47			3837 C.	8 20 47		
12	18620 G.	12 18 5	134·4	−71·1	18621 G.	12 28 10	134·4	−71·3
13	3840 C.	7 20 42	135·3	−71·1	3841 C.	7 21 43	135·6	−69·9
14	3842 C.	7 56 7	118·3	−74·6	3843 C.	7 57 7	119·3	−74·6
March 9	3832 C.	7 50 42	137·3	−62·5	3833 C.	7 51 32	136·9	−62·6
10	3834 C.	8 10 28	125·6	−63·1	3835 C.	8 11 28	125·6	−63·0
11	3836 C.	8 19 47	126·8	−62·3	3837 C.	8 20 47	126·5	−62·5
12	3838 C.	10 55 1	130·5	−63·8	3839 C.	10 57 53	131·2	−63·6
12	18620 G.	12 18 5	130·5	−63·4	18621 G.	12 28 10	130·3	−63·0
13	3840 C.	7 20 42	123·0	−63·5	3841 C.	7 21 43	122·3	−63·1
14	3842 C.	7 56 7	119·9	−63·5	3843 C.	7 57 7	120·0	−63·3
March 13	3840 C.	7 20 42	138·6	−65·4	3841 C.	7 21 43	137·7	−65·1
14	3842 C.	7 56 7	133·3	−65·9	3843 C.	7 57 7	133·0	−66·6
March 13	3840 C.	7 20 42	124·3	−64·8	3841 C.	7 21 43	124·6	−64·7
14	3842 C.	7 56 7	108·3	−64·4	3843 C.	7 57 7	107·9	−64·0

Note.—The title "veiled spots," given by M. Trouvelet to certain faint solar markings in high latitudes, may have been applied by him to markings of the same kind as these "flecks," but if so was not, I think, an appropriate one. The "flecks" as shown on the photographs do not, to my eyes, suggest "veiling"; some are faint, others dark, but none appear as if seen through an obscuring medium.

Observations of Schaumasse's Comet (b 1917) made at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The observations were made with the guiding telescope (aperture 10 inches) of the Astrographic equatorial. The driving clock was running, and the comet and comparison star were bisected by means of the slides, which have motions in the directions of Right Ascension and Declination.

G.M.T.	Comet − *. R.A.	Log Factor of Parallax.	Comet − *. Decl.	Log Factor of Parallax.
1917 h m s	s		′ ″	
May 28 10 24 36	(+ 3·45)	9·7140	+3 0·4	0·7681
10 32 10	+ 7·68	9·7093	+2 5·0	0·7753
10 36 7	+10·43	9·7067	+1 35·3	0·7800
10 40 29	+16·19	9·7036	+0 48·1	0·7852
10 48 3	+22·19	9·6977	−0 8·0	0·7941
10 55 19	+29·06	9·6915	−1 3·4	0·8022

The star is B.D.+47° 1481.

R.A. 1917·0.	Red. to App. Place.	N. Dec. 1917·0.	Red. to App. Place.
h m s	s	° ′ ″	″
7 39 6·64	+1·84	47 26 25·66	+4·07

Authority, *Bonn A.G. Catalogue*, 1875.
The observations were made by Mr. Hollis.

Royal Observatory, Greenwich:
1917 *June* 7.

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

The following measures were made at Headley, Surrey, during 1916, with the 8-inch Coleman telescope, lent by the Society. Special attention was given to (1) ξ Böotis, the *comes* of which has shown very rapid motion during recent years, and (2) to 70 Ophiuchi; in the latter case by special request.

Star.	1910. R.A.	1910. Dec.	Mags.	Position-Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 60 (η Cassiop.)	0 44	+57 20	4·0, 7·6	252·2	6·78	3	1916·983
205 (γ^2 Androm.)	1 58	41 54	5·5, 6·5	111·9	0·52	3	·978
1110 (Castor)	7 29	32 6	2·7, 3·7	216·6	5·02	3	·975
1196 (ζ Cancri) AB	8 7	17 55	5·0, 5·5	301·6	0·95	3	·296
AC	...	...	...	113·2	4·85	3	·296

Star.	1910. R.A.	1910. Dec.	Mags.	Position-Angle.	Dis-tance.	No. of Nights.	Date.
	h m	° ′		°	″		
1563 (ω Leonis)	9 24	+ 9 26	6·2, 6·7	125·1	0·94	3	1916·265
1424 (γ Leonis)	10 15	20 18	2·0, 3·5	118·0	3·89	4	·240
1536 (ι Leonis)	11 19	+11 2	3·9, 7·1	41·8	1·87	4	·288
1670 (γ Virginis)	12 37	− 0 57	3·3, 3·3	324·6	5·76	4	·359
1687 (35 Comæ Ber.)	12 49	+21 44	5·2, 8·2	91·7	0·98	3	·344
1865 (ζ Böotis)	14 37	14 6	3·5, 3·9	138·4	0·82	3	·249
1877 (ε Böotis)	14 41	27 27	3·0, 6·3	332·1	2·97	3	·249
1888 (ξ Böotis)	14 47	19 29	4·7, 6·6	94·2	2·25	4	·233
	...	...	...	92·5	2·15	6	·587
	Mean of all measures :—			93·4	2·19	10	·445
1937 (η Coronæ)	15 19	30 37	5·5, 6·0	59·2	0·88	3	·252
1938 (μ^2 Böotis)	15 21	37 40	6·7, 7·3	50·7	1·28	3	·310
2032 (σ Coronæ)	16 11	34 4	5·0, 6·1	218·3	4·82	3	·820
2055 (λ Ophiuchi)	16 26	2 11	4·0, 6·1	70·3	1·05	3	·295
2130 (μ Draconis)	17 3	54 36	5·0, 5·1	130·4	2·16	3	·383
Krueger 46	17 18	58 37	8·8, 9·0	64·8	1·50	3	·383
Σ 2199	17 37	55 48	7·2, 7·8	82·1	1·58	3	·383
2207	17 37	67 10	8·0, 8·5	124·8	0·77	3	·387
2218	17 39	63 43	6·5, 7·7	338·6	2·00	3	·387
2272 (70 Ophiuchi)	18 1	2 32	4·5, 6·0	139·2	4·78	5	·384
				138·6	4·75	6	·559
				138·3	4·70	5	·632
				136·9	4·61	5	·780
	Mean of all measures :—			138·3	4·71	21	·585
2278 BC	18 1	56 26	6·8, 7·3	146·9	5·93	3	·407
2284	18 1	65 57	7·6, 9·2	193·2	3·51	3	·387
Hussey 926	18 9	64 13	9·1, 9·3	267·8	4·03	3	·430
Σ 2307	18 12	69 13	8·5, 8·5	204·4	4·39	3	·408
2368	18 36	52 16	7·2, 7·4	326·8	2·00	3	·422
2384	18 38	67 2	8·0, 8·5	306·8	0·73	3	·400
2410	18 45	59 14	8·2, 8·7	92·5	1·44	2	·433
Hussey 938	18 54	64 25	9·1, 9·5	217·3	1·19	2	·518
Σ 2438	18 56	58 6	7·0, 7·6	17·5	0·58	3	·518
2478	19 2	69 19	8·8, 8·8	295·2	1·01	3	·590
OΣ 369	19 8	71 56	7·0, 7·3	31·1	0·72	2	·561
Σ 2509	19 16	63 2	7·0, 8·1	335·6	1·14	3	·568
β Cygni	19 27	27 46	3·0, 5·3	54·5	34·52	3	·781
Σ 2550	19 29	73 10	8·2, 8·2	249·1	1·95	2	·588

Notes.

Σ 1110 (Castor). This pair appears to be now closing.
Σ 1888 (ξ Böotis). The *comes* continues its rapid motion. The angle diminishes about a degree in six weeks, but the distance has shown little change during the last few years.
Σ 2272 (70 Ophiuchi). The four groups of measures show a small progressive diminution both in angle and distance. The former is doubtless real in part, but the distance should be slowly increasing. Unless there is a disturbing dark companion, as has been suggested (see vol. lvi., *R.A.S. Memoirs*, p. 519), it would seem that there is some evidence here of a systematic error perhaps depending on the changing slant of the line joining the two stars before and after "opposition." The possibility of such an error seems worth further investigation.
Σ 2478. Stars seem fainter than magnitudes assigned.

Errata in the Rev. T. E. Espin's Recent List of New Double Stars.

By Eric Doolittle.

This list, comprising the numbers Espin 1480 to Espin 1600, has just been published in the *Monthly Notices*, vol. lxxvii. pp. 239 to 242. In reducing the positions to the standard epoch 1880·0, and comparing the measures with the manuscript extension of Burnham's General Catalogue, the following notes and corrections are suggested :—

All of the columns are headed with the epoch 1920·0; this should be 1900·0.
Espin 1496. The R.A. is 15 sec. too small.
1499*a*. The R.A. is 20 sec. too large.
1505. The Decl. is 6 min. too small.
1509. The R.A. is 12 sec. too small.
1514. The Decl. is 2 min. too great.
1515. Jonckheere 890 must be near, but the descriptions of the two pairs differ.
1519. The R.A. is 25 sec. too small.
1537. The Decl. is 1 min. too small.
1548. The Decl. is 1·3 min. too large.
1552. Identified by Espin with B.D. +44°, 2450, but if this is correct the R.A. given is 8 min. too great and the Decl. is 9 min. too great. The place given practically agrees with that of B.D. +44° 2470, and it is assumed that this is the pair observed.
1565*a*. This pair is Roe 40. Measured by Roe in 1910 (*A.N.*, 4467).

Espin 1574. Called B.D. +41°, 3922 by Espin, but this is a 6·5 magnitude star, 6s following and 45′ north of the place given. This place, however, practically agrees with that of B.D. +41°, 3921, and it is here assumed that this was the star observed.

1575. The Decl. is 1·5 min. too small.

1576. The Decl. is 1·5 min. too small.

1582*a*. Measured by Fox in 1912 (*Annals, Dearborn Obs.*, vol. i.).

1586. The Decl. is 3 min. too small.

1592. The Decl. is 9 min. too small.

Page 236. B.D. +44°, 4550. This measure was published in *M.N.*, lxxv. p. 203, where a nearer companion was also given.

Page 238. B.D. +44°, 3552. Measures made on three nights in 1908 will be found in *M.N.*, lxix. p. 224.

The Flower Observatory:
1917 *May* 5.

Observations of SX Aurigæ. By C. Martin and H. C. Plummer. (Plate 13.)

1. The variability of SX Aurigæ was discovered in 1907 by Miss Leavitt from Harvard photographs.* Other designations are 79·1907, 050442 on the Harvard system, B.D. +41°·1101 and A.G. (Bonn) 4192. The range found by Miss Leavitt is 7·4–8·0.

2. The star has been observed by S. Enebo.† His results, based on very numerous observations, are singularly conflicting. Thus for the period he has mentioned 1·533, 1·53165, 1·53172, 1·5323, 1·53186 days. There is no doubt that the determination of the period presents a problem which has not yet found a satisfactory solution. From the usual point of view it may be said that the period is variable. But the material does not exist by which the character of the changes can be decided. It is equally possible that the changes should be attributed to more or less irregular fluctuations superposed upon a regular variation within a fairly definite period. That is a question which cannot be easily answered. The present photographic observations have been reduced with the period 1·53234 days. This serves to reconcile the observations of 1914–15 and 1915–16. It has no other foundation, though it resembles one of the periods mentioned by Enebo, and it leaves anomalies still outstanding.

3. The photographic range of magnitude, based on Schwarzschild's scale, is about 7·8–8·6. According to Miss Leavitt, the

* *H.C.O. Circular*, No. 130; *A.N.*, 4196.
† *A.N.*, 4238; *Beob. auf Dombaas*, iii., iv., vi.

star is still brighter photographically. For the visual range Enebo has given 8·0–8·7, but, after a revision of the magnitudes of the comparison stars, 8·4–9·3. The basis of this revision does not seem to be clearly indicated. But everything points to the inference that the colour index of the star is negative, perhaps to the extent of half a magnitude. This implies that the type is B–A, and with the period is consistent with the supposition that the star belongs to the same class as β Lyræ. One is then tempted, in view of the abnormal character of the light curve and the palpable irregularities to which it is subject, to see here evidence of a binary system in which the fission has occurred comparatively recently and perhaps even incompletely as yet. In any case, SX Aurigæ seems to present features of exceptional interest, and to deserve accurate photometric study. That method is obviously better suited to a case like this than the photographic method pursued in the (at present) exceptionally unfavourable climate of Dunsink. It has taken three seasons to secure 94 exposures on this one object, and the only outcome is to suggest some features of the problem without offering any solution.

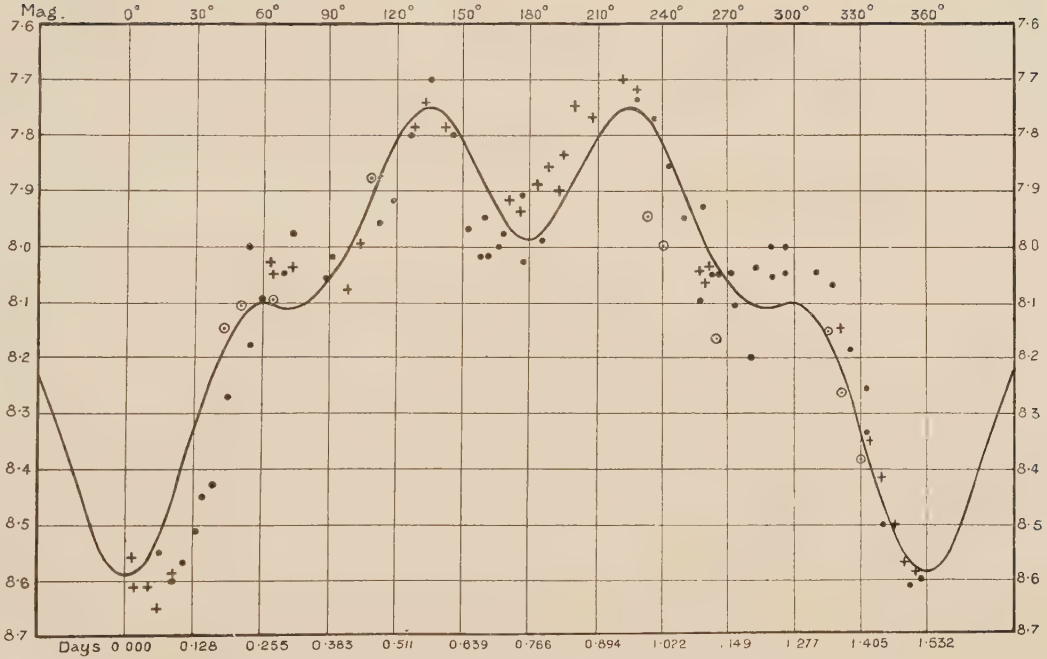
TABLE I.

B.D.	Pl. 1401. m_1.	Pl. 1434. $m_{2\cdot4}$.	Mean. m.	Corr.	Final. m'.	P.E.	Range.
+42°1197	7·70	7·67	7·69	+·001	7·69	±·014	·11
+42·1193	7·73	7·76	7·74	−·006	7·73	±·014	·11
+42·1180	8·27	8·32	8·29	+·006	8·30	±·021	·16
+41·1088	8·29	8·32	8·31	·000	8·31	±·018	·19
+41·1077	8·68	8·76	8·72	−·005	8·72	±·021	·18

4. The details relating to the comparison stars are given in Table I. They are derived from two plates exposed both on the region of the variable and on the χ Persei cluster. The magnitude scale of these plates is based on the magnitudes given by Schwarzschild for the stars in the latter region. The mean magnitudes m have been used in the reductions, though the residuals obtained from all the plates furnish mean corrections, and hence a final magnitude m' for each star on a system which should be slightly more consistent. In the last two columns of the table are also given the probable errors for a single plate, roughly estimated, and the extreme range of the largest (positive and negative) residuals. The method of reduction is that which has been explained in previous papers.

5. Table II. presents the observations on the variable. The date with G.M.T. for the middle of each exposure is given in the second column. The third column contains the corresponding Julian day and decimal fraction. The phase in the fourth column is reckoned from the epoch J.D. 2420 447·420, which is approximately half way between two principal minima, and based on the assumed period 1·53234 days. The fifth column shows the time

PHOTOGRAPHIC LIGHT CURVE OF SX AURIGÆ



Observations of 1914-15 denoted by •; of 1915-16 by +; and of 1916-17 by ⊙
Epoch of figure J.D.2420446·654; period 1·53234 days.

of exposure, and the sixth the deduced photographic magnitude of SX Aurigæ. In the last column are recorded the measurer's estimates of the quality of the images.

6. In the seventh column of Table II. are given the deviations of the observed magnitudes from the curve shown in the diagram which illustrates this paper (Plate 13). But it is not pretended that any value attaches to them. The curve represents the formula

$$m = 8{\cdot}07 - 0{\cdot}30 \cos\theta + 0{\cdot}11 \cos 2\theta + 0{\cdot}11 \cos 4\theta,$$

where, P being the assumed period, $P\theta/2\pi$ is the phase as defined in § 5. This formula was obtained as the simplest way of interpolating a symmetrical curve among the observations. The latter do not show this symmetry, as the figure indicates, but as they do not define any precise curve, the representation may as well be simple, since it cannot be accurate. The deviations are, therefore, due partly to ignoring the asymmetry in the curve, but there are other irregularities which cannot be explained in this way. Nor can they be attributed to errors of observation. The reduction to one common period is probably responsible to some extent, but the present material is too meagre for any other treatment.

7. Beyond directing further attention to a star which seems to be peculiarly interesting, this note should serve to suggest the necessary conditions for a successful attack on the problem. The means of observation must be accurate and fairly rapid. The kind of accuracy which depends on multiplying observations of low individual precision is useless here. A good light curve must be determined within the limits of as few consecutive periods as possible. This presupposes a good climate. It will then be necessary to repeat the work in successive intervals, and a comparison of the results may throw some light on the true behaviour of the star.

TABLE II.

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1914	h	m	2420		min.			
1	Nov. 10	9	45	·447·407	1·519	30	7·91	−·07	Fair.
2	11	7	51	448·328	0·908	30	8·45	+·15	Fair.
3	11	9	58	448·416	0·996	30	8·18	+·06	Fair.
4	11	10	30	448·438	1·018	30	8·10	·00	Fair.
5	11	11	24	448·475	1·055	30	8·05	−·06	Good.
6	11	11	55	448·496	1·076	30	7·98	−·13	Good.
7	16	9	38	453·402	1·385	30	7·80	+·02	Fair.
8	16	11	1	453·459	1·442	30	7·95	+·07	Fair.
9	16	11	43	453·489	1·472	30	8·00	+·06	Fair.
10	18	10	35	455·449	0·367	30	8·05	+·01	Fair.
11	18	11	16	455·469	0·387	30	8·05	−·02	Fair.
12	18	13	13	455·551	0·469	30	8·06	−·05	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1914	h	m	2420		min.			
13	Nov. 18	13	44	455·574	0·492	30	8·05	− ·06	Fair.
14	26	11	25	463·476	0·733	30	8·61	+ ·04	Good.
15	26	11	56	463·497	0·754	19	8·60	+ ·01	Fair.
16	Dec. 7	7	6	474·296	0·826	30	8·55	+ ·03	Good.
17	7	7	38	474·319	0·849	30	8·60	+ ·15	Good.
18	7	8	17	474·345	0·875	30	8·57	+ ·19	Good.
19	7	8	49	474·367	0·897	30	8·51	+ ·19	Good.
20	7	9	33	474·398	0·928	30	8·43	+ ·18	Fair.
21	7	10	14	474·428	0·958	30	8·27	+ ·10	Good.
22	8	7	56	475·331	0·329	40	7·93	− ·04	Good.
23	8	8	39	475·361	0·359	40	8·05	+ ·02	Fair.
24	8	9	32	475·397	0·395	40	8·11	+ ·04	Fair.
25	8	10	11	475·424	0·422	30	8·20	+ ·10	Fair.
26	9	7	19	476·306	1·304	33	7·80	+ ·03	Good.
27	9	8	18	476·346	1·344	22	7·70	− ·05	Fair.
28	9	9	58	476·416	1·414	35	7·97	+ ·14	Fair.
29	9	10	35	476·442	1·440	35	8·02	+ ·14	Fair.
30	18	7	0	485·292	1·096	40	8·00	− ·12	Fair.
31	18	8	25	485·351	1·155	35	8·02	− ·04	Fair.
32	18	10	35	485·442	1·246	35	7·96	+ ·08	Fair.
33	18	11	12	485·466	1·270	35	7·92	+ ·10	Fair.
	1915								
34	Jan. 8	7	4	506·295	0·646	40	8·26	− ·09	Fair.
35	11	6	30	509·268	0·554	40	8·05	− ·09	Good.
36	11	7	12	509·300	0·586	40	8·07	− ·13	Fair.
37	11	8	1	509·334	0·620	40	8·19	− ·09	Fair.
38	11	8	42	509·363	0·649	40	8·34	− ·02	Fair.
39	11	9	32	509·397	0·683	40	8·50	+ ·05	Good.
40	17	6	37	515·276	0·433	40	8·04	− ·06	Fair.
41	17	7	19	515·306	0·463	40	8·00	− ·11	Fair.
42	17	8	6	515·337	0·494	40	8·00	− ·11	Fair.
43	21	8	30	519·354	1·446	38	8·02	+ ·13	Good.
44	21	9	20	519·389	1·481	40	7·98	+ ·03	Fair.
45	21	10	16	519·428	1·520	40	8·05	+ ·06	Fair.
46	21	11	5	519·462	0·022	40	7·99	+ ·01	Fair.
47	Feb. 2	7	29	531·313	1·147	40	8·06	− ·01	Fair.
48	Mar. 19	8	17	576·345	0·208	36	7·74	− ·02	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1915	h	m	2420		min.			
49	Mar. 19	8	58	576·374	0·237	36	7·78	·00	Fair.
50	19	9	42	576·407	0·270	36	7·76	− ·07	Fair.
51	19	10	25	576·435	0·298	36	7·95	+ ·04	Fair.
52	19	11	11	576·465	0·328	34	8·10	+ ·13	Fair.
53	Nov. 2	10	3	804·419	1·496	30	7·92	− ·04	Fair.
54	2	10	37	804·443	1·520	30	7·94	− ·04	Fair.
55	2	11	19	804·472	0·017	30	7·89	− ·09	Fair.
56	2	11	50	804·493	0·038	30	7·86	− ·10	Fair.
57	2	12	29	804·521	0·066	30	7·84	− ·09	Fair.
58	2	13	0	804·542	0·087	30	7·75	− ·14	Fair.
59	2	13	47	804·575	0·120	30	7·77	− ·06	Good.
60	29	12	57	831·540	1·035	25	8·03	− ·07	Fair.
61	29	13	28	831·562	1·057	25	8·05	− ·06	Fair.
62	29	13	59	831·583	1·078	25	8·04	− ·07	Fair.
63	30	8	48	832·367	0·330	25	8·05	+ ·07	Fair.
64	30	9	13	832·384	0·347	25	8·04	+ ·03	Fair.
65	Dec. 6	12	12	838·508	0·341	17	8·07	+ ·07	Good.
66	7	8	26	839·351	1·184	30	8·08	+ ·07	Fair.
67	7	8	57	839·373	1·206	30	8·00	+ ·03	Good.
68	8	7	11	840·299	0·600	30	8·15	− ·08	Good.
69	8	8	25	840·351	0·652	30	8·35	− ·01	Good.
70	8	9	1	840·376	0·677	30	8·42	− ·02	Fair.
71	8	9	32	840·397	0·698	30	8·50	·00	Good.
72	8	10	10	840·424	0·725	30	8·57	+ ·02	Good.
73	8	10	46	840·449	0·750	30	8·59	+ ·01	Good.
74	8	11	24	840·475	0·776	30	8·56	− ·02	Bad.
	1916								
75	Jan. 27	10	43	890·448	0·182	30	7·70	− ·05	Fair.
76	27	11	23	890·474	0·208	30	7·72	− ·04	Fair.
77	Feb. 8	7	16	902·303	1·310	30	7·79	+ ·02	Fair.
78	8	7	53	902·329	1·336	30	7·75	·00	Bad.
79	8	8	45	902·365	1·372	30	7·79	+ ·02	Bad.
80	9	8	26	903·351	0·826	30	8·65	+ ·13	Fair.
81	9	9	4	903·378	0·853	30	8·59	+ ·15	Fair.
82	22	9	1	916·376	0·060	30	7·90	− ·04	Fair.
83	Mar. 6	8	35	929·358	0·783	30	8·61	+ ·03	Good.
84	6	9	13	929·384	0·809	30	8·61	+ ·06	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1916	h	m	2421		min.			
85	Nov. 24	8	44	192·365	0·228	40	7·95	+·18	Fair.
86	24	9	34	192·399	0·262	40	8·00	+·18	Good.
87	24	11	58	192·499	0·362	40	8·17	+·14	Fair.
88	25	8	46	193·365	1·228	27	7·88	−·04	Fair.
89	Dec. 22	6	58	220·291	0·572	36	8·16	−·01	Fair.
90	22	7	41	220·321	0·602	39	8·27	+·03	Fair.
91	22	8	27	220·352	0·633	36	8·39	+·08	Fair.
92	27	8	21	225·347	1·038	31	8·10	−·01	Fair.
	1917.								
93	Mar. 12	8	24	300·350	0·949	36	8·15	−·04	Good.
94	12	9	9	300·381	0·980	36	8·11	−·03	Good.